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Research Article

Demetriou's tests and levels of algebraic abilities and proportional reasoning in seventh, eighth, and ninth grades

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ABSTRACT

Developing algebraic thinking is a key factor in learning mathematics. Despite its importance, many students still struggle with algebraic concepts. This research investigates students' achievements in algebraic thinking using Demetriou's test across 7th (approximately 12-13 years old), 8th (approximately 13-14 years old), and 9th (approximately 14-15 years old) grades. The study analyzes performance in different levels of algebraic tasks (i.e., [1] extrapolation of relationships, [2] coordinating simple structures, [3] operating with undefined symbolic structures, and [4] coordination with undefined structures), revealing a significant developmental leap in algebraic abilities during the 9th grade. While no statistically significant differences were found in the first level, 9th grade students demonstrated superior performance in levels 2, 3, and 4, suggesting cognitive readiness for abstract algebraic concepts around the age of 14. The research unveils a disjointed development in algebraic abilities, indicating a progression from basic arithmetic operations to proportional reasoning before the full integration of algebraic thinking. Notably, tasks involving variables in the third level pose persistent challenges for students. The findings contribute to understanding the optimal age for introducing algebraic concepts and underscore the importance of considering cognitive development in mathematics education. The study proposes implications for educators, such as emphasizing proportional reasoning in earlier grades and employing differentiated instruction based on individual students' abilities.

Keywords: algebra, mathematics, primary school, proportions, reasoning

INTRODUCTION

The recent results of the 2022 *program for international student assessment* (PISA) international standardized mathematics tests have shown that, overall, the students' level of mathematics literacy has decreased worldwide (OECD, 2023). Although Slovenia has achieved the 12th place worldwide in students' proficiency in mathematics (OECD, 2023, p. 90), the results of OECD-PISA research show an overall decrease in students' mathematical abilities. The results of PISA often reveal a strong correlation between students' proficiency in algebraic thinking and their overall academic success (Yanto et al., 2022). Students with higher scores in algebra tend to demonstrate greater proficiency in problem-solving and critical thinking skills, reflecting a robust foundation in mathematical reasoning essential for success in various fields. There might be several reasons contributing to the lowering of students' proficiency, such as the COVID-19 pandemic; however, the structure of the national mathematics syllabus and/or programs should also be considered considering possible changes. The aim of the present research is to shed light on students' algebraic thinking abilities, especially by investigating at which age it would be more beneficial to introduce students to polynomials, variables, equations, inequalities, proportions, and algebra in general.

The development of mental representations and the understanding of mathematical concepts and facts are crucial for comprehending mathematics and constructing knowledge (Arnoux & Finkel, 2010). Concepts are objects, situations, or properties characterized by a common feature and designated by an agreed-upon symbol or sign in each culture (Fischbein, 1996). The acquisition of concepts with understanding is influenced by the developmental stage of thinking and social interaction with others (Ojose, 2008; Supratman, 2013; Tall, 2007). In our research, we examined students' cognitive development, considering Piaget's insights (Carey et al., 2015), who argued that knowledge is built on a personal level and emphasized the coherence of a student's cognitive development with the introduction of challenging concepts. Particularly in primary school, understanding the cognitive development of students is crucial, including knowledge of students' cognitive abilities necessary for understanding specific learning material and understanding ways to encourage these abilities (Vandenbroucke et al., 2018). In learning new mathematical concepts, factors such as the student's developmental stage are important (Sheromova et al., 2020), along with other factors such as prior knowledge, the teacher's introduction of new concepts, and the encouragement of processes that arise during mathematical thinking.

In recent years, literature has extensively examined the possibility of introducing children to algebra from early ages, known as the "early algebra" approach (Carraher et al., 2008; Kieran et al., 2016). Some studies have shown that the early teaching of algebra can commence as early as five years old up to 12 years old, emphasizing the importance of abstracting and creating representational modeling tools with carefully designed learning activities (Freiman & Fellus, 2021; Kim, 2013; Powell & Fuchs, 2014). Generally, algebra is introduced in school programs before secondary schools (Wilkie, 2016); many national programs and syllabuses include algebra in grade 6 (Kaur, 2014), i.e., in middle school. Instruction in algebraic concepts focuses on ratios and proportional relationships in the middle grades (Bryant & Bryant, 2016), rules for transforming and solving equations, variables, and functions (Bednarz et al., 1996). Additionally, other research has shown that students in grade 6 can learn to use their understanding of procedures and the structure of expressions in algebraic contexts (Banerjee & Subramaniam, 2012). However, students from grade 2 to grade 6 strengthen their understanding of arithmetic operations and develop ideas for the study of algebra (Russell et al., 2011).

In Slovenia, primary school children (i.e., from age 6 to age 15) are introduced to algebra from the very beginning; however, proper algebraic thinking is formally introduced in grade 7 (age 12-13). This includes the introduction of variables, monomials, and polynomials, as well as the calculation of expressions involving variables, computing with monomials and polynomials (sum, difference, product), computing with algebraic fractions, simplification of algebraic fractions, solving equations, and performing computations with functions (Žakelj et al., 2011). Nevertheless, research in Slovenia has shown that primary schools still have non-negligible difficulties in some algebraic topics (Kolar et al., 2018). Therefore, it seems important to investigate at what stage of primary school it is more efficacious to introduce students to algebra. To do so, i.e., to assess the level of formal logical thinking within mathematical concepts, Demetriou's et al. (1991) tests are often used, which are a very good indicator for determining the level of formal logical thinking, algebraic abilities, and proportional reasoning in students. Thus, the present research aims to investigate the relationship between formal algebraic thinking and age/school grade. By using Demetriou's et al. (1991) test, we aimed to verify whether the introduction of algebraic ideas in grade 7 of primary school might be appropriate and effective (cf. Žakelj et al., 2011).

MATERIALS & METHODS

Aims of Research

The development of mental representations and understanding mathematical concepts and facts is essential for constructing knowledge in understanding mathematics. In learning new mathematical concepts (Rugelj, 1996), it is crucial to consider the students' prior knowledge, how the teacher conveys new concepts, how they encourage processes that occur during mathematical thinking, and their cognitive abilities. Piciga (1995) notes that teachers often lack knowledge about the cognitive development of students, such as understanding the cognitive abilities required for understanding specific learning material, and insufficient knowledge about ways to encourage these abilities. Our research focused on this area.

We considered the findings of research on the development of cognitive abilities and recent insights influencing the understanding of mathematical concepts in connection with the development of proportional reasoning and algebraic abilities (Demetriou et al., 1991). Often, in mathematics teaching, we do not consider the cognitive development of students, leading to significant difficulties for students in understanding certain content or concepts. This most often occurs with algebra, which is highly abstract content.

Therefore, we aimed to determine at what age students are at the stage of formal logical operations. Algebraic abilities and proportional reasoning develop at this stage. It is crucial to consider this in writing the mathematics curriculum. Often, the curriculum includes content that is not suitable for the student's developmental stage. The purpose of our research was specifically for reviewing the mathematics curriculum for primary schools (Žakelj et al., 2011).

The research hypotheses were therefore the following:

H1. Among 7th, 8th, and 9th grade students, there will be significant statistical differences in knowledge on Demetriou's test, which assessed formal algebraic abilities in favor of 9th grade students.

H2. Among 7th, 8th, and 9th grade students, there will be significant statistical differences in knowledge on Demetriou's test, which assessed proportional reasoning in favor of 9th grade students.

Methodology

In the present research, the non-experimental causal research method was applied. The nature of the study is quantitative.

Sample

We included classes from the 7th (approximately 12-13 years old), 8th (approximately 13-14 years old), and 9th (approximately 14-15 years old) grades of seven different Slovenian elementary schools. The sample consisted of n=264 students from the 7th (n=89), 8th (n=101), and 9th grades (n=74). From the schools' records, the socio-economic status of the students was mainly middle.

Materials

In the present research, the Demetriou et al. (1991) test was applied. With the achievement tests designed by Demetriou et al. (1991), we assessed the level of formal-logical thinking within mathematical concepts in individuals. In the research, we analyzed students' achievements on Demetriou's test regarding their age.

Demetriou et al. (1991) created four tests that can determine the level of cognitive system development and understanding of mathematical concepts. In our research, we used two of these tests and compared the success in solving knowledge tests with the results on Demetriou's tests. We used:

(1) a test to determine the level of formal-logical thinking and algebraic abilities and

(2) a test to determine the level of formal-logical thinking and proportional reasoning.

The test to determine the level of formal-logical thinking and algebraic abilities contains 12 tasks categorized into four levels. Tasks at the first level required simple extrapolation of relationships between given elements (e.g., "If $m+n=43$, then $m+n+2=$_____"). Second-level tasks involved coordinating two simple structures (e.g., "Let $u=f+3$. If $f=1$, then $u=$_____"). Third-level tasks required the student to operate with undefined symbolic structures (e.g., "If $r=s+t$ and $r+s+t=30$, then $r=$_____"). Tasks at the fourth level demanded the coordination of undefined structures (e.g., "When is $L+M+N=L+P+N$ true?"). According to Küchemann (1981), the first two levels correspond to Piaget's early and late concrete operational stages, while the latter two correspond to his early and late formal operational stages. The test tasks are of the open-ended type, and since the tasks were brief, we monitored only the correctness or incorrectness of the answers.

Each correctly solved task was worth one point. If the answer was incorrect or unclear, the solver received zero points. The maximum number of points on the test was 12, while the minimum was zero.

The test to determine the level of formal-logical thinking and proportional reasoning contains 15 tasks categorized into four levels. All tasks are structured, as follows: "We mix paint and thinner in two containers in each ratio. In which container will the solution be redder?" First-level tasks involved mixing equal amounts of paint and thinner (e.g., "in the first container, we mix two pots of red paint and two pots of thinner, and in the second container, we mix three pots of red paint and three pots of thinner. In which container will the solution be redder?"). Second-level tasks involved mixing thinner and paint in both containers in the same ratio (e.g., "In container A, we mix two pots of red paint and three pots of thinner, and in container B, we mix four pots of red paint and six pots of thinner. In which container will the solution be redder?"). Third-level tasks related to the amount of paint in one container being a multiple of the amount in the other (e.g., "in the first container, we mix two pots of red paint and five pots of thinner, and in the second container, we mix six pots of red paint and eight pots of thinner. In which container will the solution be redder?"). Fourth-level tasks required the use of common denominators to solve (e.g., "In the first container, we mix three pots of paint and five pots of thinner, and in the second container, we mix five pots of paint and eight pots of thinner. In which container will the solution be redder?"). The first and second levels correspond to Piaget's early and late concrete operations stages, while the latter two correspond to his early and late formal operational stages (Noelting, 1980). The tasks in the test were multiple-choice.

Each correctly solved task was worth one point. If the answer was incorrect or unclear, the solver received zero points. The maximum number of points on the test was 15, while the minimum was zero. Regarding the characteristics of the used instrument, we can infer based on research whose results were published by Demetriou et al. (1991). In these studies, the authors initially included 282 high school students. From the results gathered, they found that all four mathematical abilities (the ability to integrate the four arithmetic operations, the ability of proportional reasoning, algebraic abilities, and productive knowledge application) are based on a common quantitative-relational factor. Because they wanted to determine the structure of individual abilities, the overall result of everyone was divided into two halves. Each half represents the result that an individual achieved when solving half of the test measuring a particular ability. From partial results, they calculated the reliability of the tests using correlation coefficients. Correlations between both halves of the tests were high (average correlation: r=0.84). The correlation between tests measuring different abilities is moderately strong but significantly lower than correlation between two partial results measuring the same ability (average correlation: r=0.50). Factors of algebraic and arithmetic operations are also highly related.

The same model was tested again on a sample of 372 individuals (students from primary and secondary schools aged nine to 17). More than half of the children (235) came from families with a higher socio-economic status, around a quarter of children (107) came from working-class families, and 30 children were from rural families. The results of this study also confirmed the existence of a general factor and the high reliability of the tests. The results of the second study were entirely identical to the results of the first study.

Procedure

The Demetriou's et al. (1991) tests were given to the students involved in the research. Students had one school hour (i.e., 50 minutes) to solve the problems of both tests. Mostly, students finish assignments before the end of the school hour. During the tests, both the students' mathematics teachers and one researcher were present in the class. The research took place in March 2022.

Ethical Considerations

Prior to the research, informed consent was obtained from the students' parents or guardians. They were informed about the purpose, procedures, and potential implications of the research, ensuring voluntary participation. Confidentiality was ensured during the conduct of the study: codes were used to collect data, which was handled only by the researchers and in compliance with privacy standards. The research was conducted in accordance with the European code of conduct for research integrity (ALLEA, 2023).

Data Analysis

Data were analyzed using methods of descriptive and inferential statistics. In particular, the frequency analysis has been used to analyze the proportions of correct answers to the Demetriou's et al. (1991) tests.

Table 1. Achievements on Demetriou's test of algebraic thinking

Item	Level	7th grade (%)	8th grade (%)	9th grade (%)
If a+5=8, then a=_____.	1	100	100	95
If m+n=43, then m+n+2=_____.	1	78	78	72
2k+5k=_____.	1	81	86	83
Let u=f+3. If f=1, then u=_____.	2	78	76	89
Let m=3n+1. If n=4, then m=_____.	2	24	32	72
2a+5b+a=_____.	2	0	6	62
If e+z=8, then e+z+n=_____.	3	0	8	38
3s-v+s=_____.	3	0	4	52
If x=y+z, and x+y+z=30, then x=_____.	3	28	31	24
Compute (m+5)×4.	4	0	6	62
When does it hold: L+M+N=L+P+N?	4	17	43	35
For which n does it hold: 2n>2+n?	4	10	17	27
Total		10	41	60

Additionally, Chi-square was used to check for possible differences in the proportions of the answers divided among the three school grades involved in the research.

RESULTS

Achievements on Algebraic Thinking Test

Students' achievements on Demetriou's test of algebraic thinking are presented in **Table 1**. In **Table 1**, the percentages of correct answers distinguished among 7th, 8th, and 9th grades are presented.

An analysis of the differences in achievements among grades has shown no statistically significant difference in achievements concerning the first level (χ^2[4]=.245; p=.993), however grade 9 students achieved better in level 2 problems (χ^2[4]=67.690; p<.001), level 3 (χ^2[4]=75.000; p<.001), and level 4 problems (χ^2[4]=50.780; p<.001).

Comparison of the achievements of 7th, 8th, and 9th grade students by levels (**Table 1**) shows that there is a significant development in algebraic abilities in the 9th grade, making the introduction of abstract algebraic concepts (e.g., the concept of a variable) possible in this period from the perspective of cognitive development. However, the introduction of these concepts still needs to be tied to a connection with concrete objects. In this period, formal-logical thinking and algebraic abilities surpass the abilities of proportional reasoning, which was somewhat greater than algebraic abilities in the 7th and 8th grades (**Table 1**). Although there is a disjointed development in algebraic abilities (Demetriou et al., 1991), students first acquire the ability to calculate and use the four operations, and only when these are acquired does the development of proportional reasoning begin. Algebraic abilities develop between the blocks of computational abilities and proportional reasoning. According to Demetriou et al. (1991), the first phase of the development of proportional reasoning begins from the age of 12/13 to 14/15, and more significant development occurs during the transition to high school. The development of algebraic abilities occurs during the transition from the 8th to the 9th grade, and essential development in integrating all four arithmetic operations occurs in the 7th and 8th grades. When testing algebraic abilities, tasks of the third level caused the most difficulties for students. Not only was the performance poor in the 7th grade, but it also did not significantly increase in the 8th and 9th grades. These are tasks that our students do not often perform in school. This probably means that the concept of a variable, introduced in the third triad, is not so straightforward and self-evident.

Achievements on Proportion Test

Students' achievements on Demetriou's test of proportion thinking are presented in **Table 2**. **Table 2** reports the percentages of correct answers in grades 7, 8, and 9.

An in-depth analysis of students' achievements on the second test has shown that there are no statistically significant differences in achievements among grades concerning level 1 problems (χ^2[4]=.470; p=.986), level 2 (χ^2[4]=8.450; p=.076), level 3 (χ^2[10]=6.750; p=.749), and level 4 problems (χ^2[4]=2.420; p=.659).

More than half of 7th graders (**Table 2**) successfully solved only tasks of the first level–mixing the same amount of color and diluent. In the 8th grade, this percentage increased, and just under half of the 8th grade

Table 2. Achievements on Demetriou's test of proportion thinking

Item	Level	7th grade (%)	8th grade (%)	9th grade (%)
2:2-3:3	1	72	89	86
1:1-2:2	1	62	87	85
1:1-3:3	1	69	87	85
2:4-3:6	2	22	50	45
1:2-2:4	2	32	41	68
4:2-2:1	2	38	48	49
2:1-4:3	3	41	33	44
2:3-3:4	3	82	91	88
2:3-1:2	3	21	24	29
1:3-2:5	3	29	22	35
6:3-5:2	3	33	39	37
4:2-5:3	3	30	41	34
5:2-7:3	4	27	26	28
3:5-5:8	4	24	15	14
5:7-3:5	4	17	17	17
Total		14	47	50

Note. Item: We mix paint & thinner in two containers in each ratio. In which container will solution be redder?

students successfully solved tasks of the second level (mixing colors and diluents in a ratio of 1:2, 2:4). In the 9th grade, the abilities for proportional reasoning are already better developed, primarily in the first and second levels, and partly in the third level as well (the amount of color in the second container is a multiple of the amount of color in the first container). However, in the 9th grade, many students do not reach the levels of the fourth level. According to Demetriou et al. (1991), the first phase of proportional reasoning development occurs between the 7th and 8th grades, with significant development happening during the transition to high school. Obvious development occurs between the 7th and 8th grades, and the results of the 8th and 9th grades in our sample do not significantly differ, meaning that there is no substantial development during this period.

DISCUSSION & CONCLUSIONS

2022 PISA (OECD, 2023) revealed a global decline in students' overall mathematics literacy, with Slovenia ranking 12th worldwide in this regard. Despite Slovenia's commendable position, OECD-PISA research indicates a general decrease in students' mathematical abilities. Various factors may contribute to this decline, prompting a need to scrutinize the national mathematics syllabus (cf. Žaklej et al., 2011) and programs for potential adjustments. The present research focuses on shedding light on students' algebraic thinking abilities, investigating the optimal age for introducing algebra. The study aligns with the crucial role of cognitive development in comprehending mathematics and constructing knowledge (Arnoux & Finkel, 2010). It draws from Piaget's insights (Carey et al., 2015), emphasizing the coherence of cognitive development with the introduction of challenging concepts. The investigation in Slovenia considers the formal introduction of algebraic thinking in grade 7 (cf. Žakelj et al., 2011), with the study aiming to assess the relationship between formal algebraic thinking and age/school grade, utilizing Demetriou's et al. (1991) test to verify the appropriateness and effectiveness of introducing algebraic ideas at this stage. The research underscores the importance of understanding students' cognitive development and employing effective teaching approaches in mathematics education (cf. Vandenbroucke et al., 2018).

In our research, we have demonstrated that when comparing students from 7th, 8th, and 9th grades, there is a significant development in algebraic abilities in the 9th grade (around 14 years old). Therefore, the introduction of abstract algebraic concepts (such as the concept of variables, algebraic expressions, and algebraic equations) during this period is cognitively feasible (cf. Demetriou et al., 1991). However, it is still necessary to connect the introduction of these concepts to concrete objects or real-life situations (cf. Booth et al., 2015). For instance, students could encounter algebraic problems from the agriculture and farming world (Chang & Huang, 2014), sports, and technology (Kaya & Dincer, 2022). During this period of formal-logical thinking, algebraic abilities surpass proportional reasoning abilities (Bronkhorst et al., 2021), which were slightly greater in the 7th and 8th grades compared to algebraic abilities. In particular, Demetriou et al. (1991) state that level 1 proportional items are not understood before the age of 11 and 12 (i.e., grade 7).

It is important to note that not only children but also many adults are not capable of formal-logical thinking and, consequently, struggle to comprehend algebra (Manly & Ginsburg, 2010). This observation is supported by the experiences of the authors, all of whom have also taught high school students aged 15 to 19. During the study, one student, for example, expressed, "why are we working with letters? I do not understand. I would like to calculate with numbers." Another student questioned, "why do I need these concepts? What does the result 'a' mean?"

Based on theoretical knowledge of children's cognitive development, including recent insights into children's thinking and an understanding of social cognition, learning, and teaching, an appropriate didactic approach for teaching algebra should be developed. This approach should rely on the theory of developmental psychology, which examines the development of concepts based on the developmental stage of children's thinking (Warren et al., 2016), and consider recent cognitive-constructivist findings in pedagogy that emphasize the learner's activity in the learning process (cf. Cotič & Zuljan, 2009). When designing a didactic approach, it is crucial to consider that the learning process is significantly influenced by the developmental stage of thinking, the structure of existing knowledge, and the organization of learner activities or stimuli from the environment. Additionally, interpreting a child's thinking considering recent insights into metacognition and the connection between thinking and language is desirable (cf. Desoete & De Craene, 2019; Schneider & Artelt, 2010).

All these aspects are often overlooked in mathematics education, as teachers may lack sufficient knowledge in developmental psychology (cf. Lohse-Bossenz et al., 2013). Not all students can grasp abstract concepts in mathematics, and therefore, mathematics instruction should be differentiated or tailored individually to each student based on their abilities. We believe that one of the fundamental causes of the fear of mathematics is often exceeding the abstract abilities of students with content and concepts. The teacher's task is precisely to develop not only the student's knowledge but also their positive attitude towards mathematics and their positive self-image as a mathematician.

Limitations & Future Research

The present research is not without limitations. Firstly, the sample size in this study could be expanded, and future studies may explore similar research questions with a larger and more diverse sample to enhance generalizability. Secondly, although we utilized a standardized and validated instrument to measure students' algebraic thinking abilities, employing different tests could potentially yield different results. Thirdly, the present study focused solely on students' algebraic thinking skills, without considering any other variables. Future research could explore the impact of various factors, including students' motivation, mathematics anxiety, and teacher-assigned grades, on the comprehension of algebra.

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Data availability: Data generated or analyzed during this study are available from the authors on request.

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