European Journal of Science and Mathematics Education, 2025, **13**(2), 103-118
ISSN: 2301-251X (Online)

Research Article

Design and validation of a questionnaire to explore the geometric work of mathematics teachers

Carolina Henríquez-Rivas [1*]
0000-0002-4869-828X

Andrea Vergara-Gómez [1]
0000-0001-6388-8412

[1] Centro de Investigación en Educación Matemática y Estadística (CIEMAE), Facultad de Ciencias Básicas, Universidad Católica del Maule, Talca, CHILE
[*] Corresponding author: chenriquezr@ucm.cl

Citation: Henríquez-Rivas, C., & Vergara-Gómez, A. (2025). Design and validation of a questionnaire to explore the geometric work of mathematics teachers. *European Journal of Science and Mathematics Education, 13*(2), 103-118. https://doi.org/10.30935/scimath/16161

ARTICLE INFO

Received: 6 Aug 2024
Accepted: 31 Jan 2025

ABSTRACT

Although research highlights the importance of analyzing the geometric work of teachers, there are few validated instruments in this line. This study presents the processes of design and validation of a forced-choice questionnaire that allows the characterization, from a theoretical basis, of how geometry teachers organize their teaching practice. From the theoretical perspective of mathematical working spaces, dimensions and subdimensions were structured. Content validation was achieved by submitting the questionnaire for expert judgment. To analyze the internal consistency, Aiken's V coefficient and Kendall's coefficient of concordance were used. The results permitted the general structure of the instrument to be maintained. The final version of the instrument consists of 23 items organized into 3 theoretically sustained dimensions, allowing the exploration of geometry teaching practices among mathematics teachers.

Keywords: questionnaire, secondary education, geometry, teachers, mathematical working spaces

INTRODUCTION

This study focuses on exhibiting the process of design and validation of a questionnaire based on the theory of mathematical working spaces (MWS). In general, the findings support the consistency of the questionnaire for analyzing how teachers organize their geometry teaching practice. This aims to contribute to the community of researchers, teacher educators, and in-service teachers in terms of the use of the instrument to orient decision-making and feedback processes toward mathematics teacher training.

Research on geometry education is varied and extensive, and it has undergone increasing growth in recent years (Herbst et al., 2018; Jones & Tzekaki, 2016; Sinclair et al., 2017). In recent decades, diverse studies in geometry have been underpinned by different theoretical perspectives or focused on specific geometric processes (e.g., Arzarello et al., 2007; Costa & Del Río, 2019; Duval 2005; Fernández Blanco et al., 2012; Hershkowitz, 1990; Kuzniak, 2006, 2018; Prior & Torregrosa, 2020; Torregrosa & Quesada, 2007). Other research addresses the use of tools for dynamic geometric work (e.g., Arzarello et al., 2002; García López et al., 2021; Hankeln & Greefrath, 2021; Henríquez-Rivas & Kuzniak, 2021; Lagrange & Richard, 2022; Richard et al., 2019).

In recent studies, the importance of analyzing teachers' geometrical work has been highlighted, focusing on in-service teachers and pre-service teachers based on diverse theoretical perspectives and methodologies (e.g., Avcu, 2022; Ayvaz et al., 2017; Clemente & Llinares, 2015; Creager, 2022; Kuzniak & Nechache, 2021). For example, Zararyan and Sosa (2021) describe a secondary school teacher's knowledge of mathematical practice in geometry classes based on the model of mathematics teacher's specialized knowledge and conclude that there is a lack of empirical data supporting teacher training. Meanwhile, Sunzuma and Maharaj (2020) analyze the perspectives of secondary-level in-service teachers on geometry teaching and recommend teacher training using theoretical approaches.

The research of Guzmán Retamal et al. (2020) explores the classroom management of educators who teach geometrical topics, underscoring the need for studies on teachers' mathematical practices in the classroom. Likewise, Martínez-Mora and Camargo-Uribe (2021) focus on changing geometric work centered on exercises from the perspective of problem-solving, highlighting the importance of teachers' classroom management to generate changes. Another study focused on the teaching of geometry, which demonstrates the lack of mathematical knowledge and instruction practices among teachers, points to the necessity for teacher training and additional research of this type at a larger scale (Tachie, 2020). Thus, it is necessary to expand research using this type of approach, which entails the design of instruments that allow the collection of this type of data.

In terms of instrument design, validation is a fundamental process in their development, evaluation, and use, as it provides support for the suitability, significance, and utility of decisions that can be made based on the scores obtained (Chan, 2014; Kirkland et al., 2024). In the development of instruments for topics related to mathematics education in particular and their respective processes of validation, content validation is frequently carried out by expert judgement (e.g., Espinoza Salfate et al., 2023; Seguí & Alsina, 2023) and construct validation by analysis of dimensions or factors (e.g., Magaña Medina et al., 2023; Verdugo-Castro et al., 2022). Likewise, various types of instruments can be found, from observation guides (Olfos Ayarza et al., 2022) to tests for specific knowledge (e.g., Díaz, 2022).

Indeed, distinct types of instruments have been developed and validated, especially centered on mathematics teachers (e.g., Mohr-Schroeder et al., 2017; Vásquez Ortiz et al., 2020). Examples include an observation guide intended for interpreting teachers' practices in mathematics classes in secondary education, considering mathematical contents, the teaching approaches corresponding to the mathematical contents, and classroom management (Arteaga-Martínez et al., 2021); a Likert scale questionnaire to gather the opinions of future educators on the educational values of mathematics (Dede, 2011); a questionnaire with open-ended questions to evaluate the didactic-mathematical knowledge required for future primary school teachers for teaching elementary mathematics (Pincheira-Hauck & Vásquez-Ortiz, 2018); an observation guide to evaluate the practical mathematical teaching abilities of pre-service preschool teachers (Olfos et al., 2022); and an open-ended questionnaire to analyze aspects of specialized knowledge on teaching statistics and probability among in-service primary school teachers (Seguí & Alsina, 2023). While each of these instruments corresponds to a specific context and purpose, it was not possible to find an example that allowed investigating the geometry teaching practices of teachers in secondary education.

As indicated above, research on geometry teaching reflects an unresolved problem in teachers' professional development, underscoring the lack of studies that connect educators' mathematical practice for the organization of teaching with existing theoretical models (e.g., Henríquez-Rivas et al., 2021). This problem highlights the need for greater focus on geometry teaching and research of this type at a larger scale. Therefore, the present study is formulated based on the research objective of developing and validating an instrument intended to characterize, from a theoretical basis, how teachers organize their practice for the teaching of geometry. In this sense, the process of construction of the corresponding instrument and its content validation by expert judgement is demonstrated. A theoretical framework relevant to this objective is presented in the following section.

THEORETICAL FRAMEWORK

The theory of MWS initially emerged for the teaching of geometry (Kuzniak 2006), and subsequently it has been expanded to other mathematical domains (e.g., Machuca Pérez & Montoya Delgadillo, 2022; Montoya-Delgadillo & Vivier, 2016). The objective of MWS is to study the mathematical work of people who solve math problems, in addition to characterizing the routes that emerge in the process of solving them (Kuzniak et al., 2022).

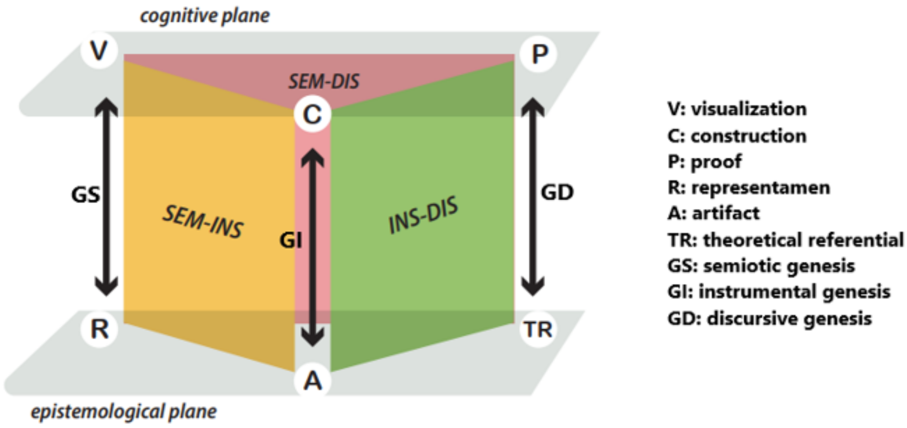

Figure 1. MWS diagram (adapted from Kuzniak et al., 2016, p. 727, reprinted with permission)

MWS considers the epistemological principles underlying the objects of study within a mathematical domain and the cognitive processes of the individual when they acquire, develop, or utilize those mathematical contents. These two dimensions are termed epistemological and cognitive planes (Kuzniak, 2011, 2022). The epistemological plane has three components: the representamen, associated with a set of concrete and tangible symbols in accordance with the interpretations and relations constructed by the individual; artifacts, such as drawing tools, software, or a symbolic system, employed as an instrument for action; and the theoretical referential, based on definitions, properties, and theorems. The cognitive plane is organized based on three processes: visualization, related to the use of signs to represent objects; construction, a process based on the actions triggered by the artifacts used and the associated usage techniques; and proof, a process understood as all of the discursive reasoning that permits formulating deductively organized arguments, definitions, hypotheses, and conjectures, as well as stating counterexamples (Kuzniak, 2022; Kuzniak et al., 2016).

The two planes are connected through semiotic, instrumental, and discursive genesis, which allow the nature of the mathematical work in diverse educational and institutional contexts to be coordinated and made explicit (Kuzniak, 2022). Semiotic genesis represents the relation between the mathematical object and the cognitive process of visualization to endow it with meaning. In instrumental genesis, artifacts are operationalized through the construction undertaken by the individual. Discursive genesis relates the referential and the processes of proof. In the solving of a given problem, the geneses and the components of the two planes interact, which is referred to as vertical planes (Kuzniak & Richard, 2014). Specifically, in these interactions, three different vertical planes are identified: the Sem-Ins vertical plane, associated with the use of artifacts in the construction of results under certain conditions or in the exploration of semiotic representations; the Ins-Dis vertical plane, linked to the process of proof based on experimentation with the use of an artifact, or the validation of a construction; and the Sem-Dis vertical plane, which involves the coordination of the process of visualization of represented objects with validation reasoning (Kuzniak et al., 2016). These relations are illustrated in the following diagram (**Figure 1**).

In this manner, MWS research is based on studying and understanding the dynamic of mathematical work through the role of each of these geneses and their interactions (Kuzniak, 2018). Thus, the MWS, with its planes, components, and geneses, is an analytical and methodological tool for identifying the different phases of the process of problem-solving and for describing the evolution of mathematical work (Kuzniak & Nechache, 2021; Nechache & Gómez-Chacón, 2022).

In the MWS theory, three types of MWS are specified based on users, position in an educational institution, and role in the implementation of the school curriculum (Gómez-Chacón et al., 2016): *referential* MWS, *personal* MWS, and *idoine* MWS. The present study is related to the idoine MWS, which is understood as the manner in which particular mathematical contents developed by a professor (or researcher) are designed, adapted, modified, and presented for teaching in a specific institution and context (Henríquez-Rivas et al., 2021, 2022). Ultimately, the MWS theory is a didactic theory that is unique among other approaches used in mathematics education today, relating mathematical contents by closely combining epistemological aspects of math with cognitive processes of the subjects (Radford, 2017). In this manner, the richness of this

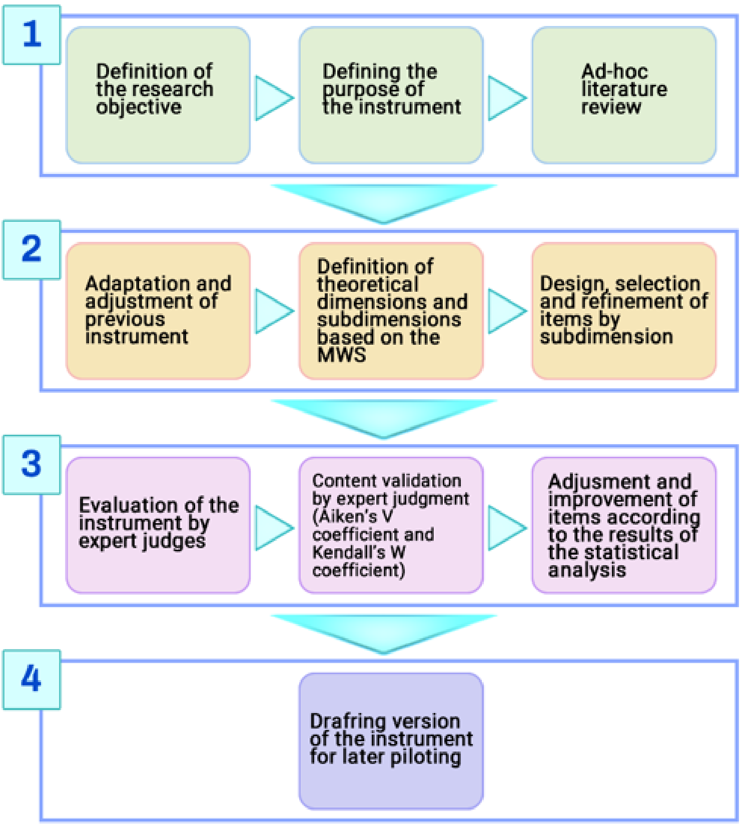

Figure 2. Workflow for the design and validation of the instrument (Source: Authors' own elaboration)

theoretical corpus has guided the design of the instrument at the center of this study, which allows aspects of teachers' geometrical work to be identified.

METHODOLOGY

This study is exploratory and descriptive in nature (Lederman, 1993), as it is focused on carrying out a primary evaluation of the characteristics and psychometric properties of the self-administered questionnaire Idoine MWS in the geometrical domain (MWS-IG questionnaire [**Appendix A**]). This process took place through content analysis by expert judgement (Escobar-Pérez & Cuervo-Martínez, 2008) using Aiken's V coefficient to analyze the internal consistency of the items and Kendall's coefficient of concordance (W) coefficient to evaluate the dimensions. The procedures utilized are presented here, firstly, in terms of the construction of the MWS-IG questionnaire, and then regarding the content validation together with the required adjustments that resulted from the validation process.

The questionnaire was developed in the framework of a wider research project that enables the identification of elements of geometric work among in-service mathematics teachers in the first year of secondary school in Chile (teaching students aged approximately 14). Specifically, the teaching topics considered within the geometric domain include Thales' theorem, homothecy, and similarity of figures. The selection of school level and teaching topics is based on the necessity to expand the existing research on geometry teaching and for it to be transferable to pre-service teacher education in mathematics. In addition, the selection of these geometric topics has been undertaken based on curricular and epistemological aspects, given that they are present both in the school curriculum (Ministerio de Educación de Chile [Ministry of Education of Chile] [Mineduc], 2015), and in the *estándares disciplinares* [disciplinary standards] of pre-service teacher education in Chile (Centro de Perfeccionamiento, Experimentación e Investigaciones Pedagógicas [Center for Improvement, Experimentation and Pedagogical Research] [CPEIP], 2021). In both cases, the study of said topics is presented as being linked, since it is approached from the perspective of the group of transformations in the Euclidean plane. Subsequently, the process of development and validation of the instrument consisted of four main stages, which are reflected in the following workflow diagram (**Figure 2**).

Table 1. Range of experience of the experts consulted

Area of experience	Total
Pre-service teacher training of math teachers	4
Continuing education of in-service math teachers	2
Both	6

In the first stage, considering the research objectives of the project, the purpose of the instrument was defined and an ad-hoc literature review was conducted. In the second stage, certain elements were adapted and adjusted from an instrument designed for situated observation using a web application, which allows analysis, with theoretical foundation in the MWS, of the classroom practice of mathematics teachers when carrying out geometry classes (Henríquez-Rivas, 2017). Concurrently, theoretical dimensions and subdimensions were defined based on the MWS, focusing on the self-administered nature of the present instrument, and each researcher independently constructed a set of questions for each subdimension. Next, the items designed were compared, selected, and adjusted according to the goal of measuring dimensions and subdimensions.

In the third stage, the instrument was submitted for evaluation by expert judgement, with the respective experts responding to a consultation protocol via email. With the data from this consultation, content validation was carried out using two statistical coefficients; the results informed the selection of items that required adjustments or improvements. Lastly, in the fourth and final stage, relying on the comments and suggestions of the expert judges, the instrument was improved, with special attention to the relegated items. In this manner, the version of the instrument to be subsequently utilized for a pilot with schoolteachers was formulated. The results of the present work focus on stages 2, 3, and 4 of the construction, content validation, and corresponding adjustments of the questionnaire.

Experts Consulted and Process of Content Validation

The call for expert judges was made following the criteria posited by Skjong and Wentworth (2004), specifically the following: previous experience providing expert judgement in this research area, reputation in the scientific community, availability and motivation to participate, impartiality, and other qualities inherent to thorough researchers in this area. In this manner, a group of 12 experts was selected (6 men and 6 women) with doctorate degrees in mathematics education or related areas, with undergraduate degrees in mathematics or mathematics education and a recognized academic trajectory with experience in mathematics teacher education in disciplinary or didactic-disciplinary courses (**Table 1**). Additionally, the judges possessed knowledge of the theoretical framework underlying the instrument. Specifically, 8 judges have carried out research directly based on MWS, and the rest are involved in research with cognitive theories related to certain theoretical aspects of MWS. All of the experts consulted had worked for at least ten years in university teaching or research in the teaching and learning of mathematics, and all held positions at Chilean universities.

For purposes of applying the scale, the experts were duly informed of the research objectives. The ethical aspects of the study were fulfilled in accordance with the requirements of the ethics committee of the university endorsing the research project. The presentation of the instrument to the expert judges included a request letter and a document containing a detailed description of the dimensions and subdimensions by which the instrument was organized, together with the instrument broken down by subdimension. Through a non-comparative Likert scale ranging from 1 to 4 (with 1 being "does not fulfill" and 4 being "completely fulfills"), the experts evaluated each item according to three criteria: *clarity*, *appropriateness*, and *relevance*. In addition, together with these three criteria, judges were asked to evaluate the *sufficiency* of the items defined for each subdimension for measuring what each subdimension was intended to measure. The Likert scale values were specified for each of these criteria. The instructions for the evaluation also indicated a section for experts to make comments at the end of each subdimension if they deemed it necessary.

Once the data was collected, the descriptive statistics were calculated for the scores obtained for each item. Subsequently, the content validity was evaluated for each item using Aiken's V coefficient (Aiken, 1985). This coefficient offers an index of validity and allows the measurement of the proportion of judges who express a positive evaluation regarding a given item, which can be adopted as a criterion for decision-making regarding retaining or eliminating items (Martin-Romera & Molina Ruiz, 2017). Aiken's V coefficient can be used as an index of content relevance (Penfield & Giacobbi, 2004) in a manner that helps guide the retention or elimination of items. To perform the calculation, Excel was used, adopting a significance level of 0.05 for the lower and upper limits and of confidence interval. This was carried out for the three criteria for each of the 23 items. To make the final decision, the Aiken's V coefficients obtained for the three criteria were combined (clarity, appropriateness, and relevance) using a Euclidean distance standardized to the interval [0, 1]. The same technique was applied to outline the lower limits of the confidence intervals for the three criteria in each of the elements. To complete the definitive selection of elements, the fulfillment of two conditions was considered under the logical connective "and":

1. A global Aiken's V coefficient of the three criteria, obtained using Euclidean distance, greater than 0.8.
2. A lower limit of the global confidence interval of the three criteria, obtained using Euclidean distance, greater than 0.6.

Along with the above, the p-value for each item was calculated with 95% confidence, considering the three criteria (clarity, appropriateness, and relevance) independently. In addition, to measure the degree of concordance and internal consistency of the variables among the experts, the non-parametric test of Kendall's W index was applied by dimension. Kendall's W coefficient is utilized to determine the degree of concordance between *k* sets of rankings, so its use is appropriate when a group of judges is asked to evaluate various items of an instrument, assigning ordinal values within a determined range. To perform the calculations, the R 3.3.0 programming language was used in the R-Studio environment version 2023.06.02+561. The analysis was carried out considering the criteria of clarity, appropriateness, and relevance by dimension to identify which of these might require adjustments and under which specific criteria. Decision-making was made assuming a significance level of 0.05 and, as a null hypothesis, that the degree of agreement among the judges was due to randomness. As a statistical criterion for the analysis, the p-value was prioritized over the Kendall's W value.

In sum, Aiken's V coefficient was used to decide whether to discard or retain each item, and Kendall's W coefficient was used to evaluate the degree of concordance based on criterion and dimension. The results of the Aiken's V coefficient for each criterion were used to determine which items required adjustments, that is, whether it was necessary to improve clarity, appropriateness, or relevance. To implement adjustments according to criteria, the experts' qualitative observations were attended to regarding how to improve each subdimension.

RESULTS

Given the absence of studies specifically focused on classroom observation instruments allowing examination of the geometry teaching practices of secondary-school teachers, which has been mentioned in previous studies (Vásquez Ortiz et al., 2020), the MWS-IG questionnaire has been designed based on the MWS theory. Below, the process of design, content validation, and corresponding adjustments made to the questionnaire will be illustrated.

Design of the Initial Version of the MWS-IG Questionnaire

For the initial design of the MWS-IG questionnaire, first a literature review was carried out, with a focus on various types of documents related to the teaching and learning of geometry; these included the following:

1. Studies on geometry education consider diverse theoretical and methodological perspectives, with a focus on in-service and pre-service teachers (e.g., Zakaryan & Sosa, 2021).
2. Searching for existing instruments in mathematics education that consider rigorous validation processes, especially those centered on the mathematics teacher (e.g., Arteaga-Martínez et al., 2021).
3. Literature review based on the MWS theory. In this instance, special attention was placed on those theoretical and empirical studies that consider the domain of geometry and the teacher as subject of the study (Kuzniak et al., 2022; Panqueban et al., 2024).
4. Certain guiding documents for teaching in Chile in pre-service teacher education, such as the national curriculum (Mineduc, 2015) and the *estándares de la profesión docente de carreras de pedagogía en matemática de Chile* [professional teaching standards of mathematics pedagogy programs in Chile] (CPEIP, 2021).

Table 2. Dimensions and subdimensions of the MWS-IG questionnaire

Dimensions (D)	Sub-dimensions (S)	n	Question type
D1. Teaching preparation	S1. Teaching organization	3	Explore the organization, preparation, and evaluation of the teaching of homothecy, Thales' theorem, and similarity of figures.
	S2. School curriculum	5	
D2. MWS components and geneses	S3. Components	6	Explore the teaching of geometry, without focusing on specific concepts or theorems.
	S4. Geneses	3	Refer to the MWS geneses in relation to the teaching of homothecy, Thales' theorem, and similarity of figures.
D3. MWS vertical planes	S5. Vertical plane [Sem-Ins]	2	Refer to the connection between two geneses associated with the teaching of homothecy, Thales' theorem, and the similarity of figures.
	S6. Vertical plane [Ins-Dis]	2	
	S7. Vertical plane [Sem-Dis]	2	

Note. n: Number of items

Table 3. Statistics for the scores assigned by the judges, for each sub-dimension and the first three criteria

Sub-dimension		S1	S2	S3	S4	S5	S6	S7
Clarity	Mean	3.361	3.517	3.305	3.389	3.583	3.458	3.500
	Kurtosis	−0.978	−0.121	−1.039	−0.589	0.609	−0.429	−0.158
	Asymmetry coefficient	−0.765	−1.191	−0.585	−0.691	−1.522	−1.054	−1.198
	Standard deviation	0.798	0.748	0.762	0.688	0.775	0.779	0.780
	Coefficient of variation	23.8%	21.3%	23.1%	20.3%	21.6%	22.5%	22.3%
Appropriateness	Mean	3.694	3.783	3.986	4.000	4.000	3.958	4.000
	Kurtosis	2.563	4.544	72.000	NA	9.123	24.000	NA
	Asymmetry coefficient	−1.924	−2.257	−8.485	NA	−3.219	−4.899	NA
	Standard deviation	0.624	0.490	0.118	0.000	0.282	0.204	0.000
	Coefficient of variation	16.9%	12.9%	2.9%	0%	7.2%	5.1%	0%
Relevance	Mean	3.833	3.967	3.972	3.972	4.000	3.958	4.000
	Kurtosis	8.822	27.360	33.384	36.000	NA	24.000	NA
	Asymmetry coefficient	−3.093	−5.333	−5.870	−6.000	NA	−4.899	NA
	Standard deviation	0.507	0.181	0.165	0.167	0.000	0.204	0.000
	Coefficient of variation	13.2%	4.6%	4.2%	4.2%	0.0%	5.1%	0.0%

5. Selected geometry texts as support and reference material, for example, Chuaqui and Riera (2011), among others.

The MWS-IG questionnaire consists of 23 multiple-choice, single-answer items. Each question presents five forced-choice options, described in a user-friendly language that reflects local teaching practices. The items are questions posed to the teacher, specifically exploring the three dimensions:

(1) teaching preparation,

(2) MWS components and geneses, and

(3) MWS vertical planes.

Each of these dimensions includes subdimensions with question types oriented toward the geometric domain, especially centered on the teaching of homothecy, Thales' theorem, and the similarity of figures. The first and the second dimension, each consider two subdimensions. The third dimension considers three subdimensions.

For the construction of the dimensions and subdimensions, theoretical elements specific to MWS were considered. On the one hand, transversal aspects of the theory are considered (e.g., MWS components, geneses, and vertical planes), which allow the mathematical work of the subject to be analyzed. On the other hand, specific aspects of the MWS theory related to teaching are considered, which are linked to the definition of the *idoine MWS* (Henríquez-Rivas et al., 2022). Details regarding the quantity of items and the general description of the questions by dimension is shown in **Table 2**.

General Analysis

For the analysis of inter-judge consistency, the mean, standard deviation, and coefficient of variation were calculated for each subdimension, considering the scores assigned by the twelve judges, with the purpose of establishing the homogeneity of the evaluations (**Table 3**).

Table 4. Results of inter-judge consistency per item for the three criteria

Item	Clarity			Appropriateness			Relevance			SAV	SLL	II
	V	LL	p	V	LL	p	V	LL	p			
1	0.78	0.64	*	0.86	0.73	*	0.89	0.77	*	0.84	0.72	Yes
2	0.78	0.64	*	0.94	0.84	*	0.97	0.88	*	0.90	0.79	Yes
3	0.80	0.67	*	0.89	0.77	*	0.97	0.88	*	0.89	0.78	Yes
4	0.86	0.73	*	0.94	0.84	*	1.00	0.92	*	0.94	0.84	Yes
5	0.86	0.73	*	0.94	0.84	*	1.00	0.92	*	0.94	0.84	Yes
6	0.86	0.73	*	0.94	0.84	*	1.00	0.92	*	0.94	0.84	Yes
7	0.78	0.64	*	0.89	0.77	*	0.94	0.84	*	0.87	0.75	Yes
8	0.83	0.70	*	0.92	0.80	*	1.00	0.92	*	0.92	0.81	Yes
9	0.69	0.55	*	1.00	0.92	*	1.00	0.92	*	0.91	0.82	Yes
10	0.78	0.64	*	1.00	0.92	*	1.00	0.92	*	0.93	0.84	Yes
11	0.78	0.64	*	1.00	0.92	*	1.00	0.92	*	0.93	0.84	Yes
12	0.67	0.52	> 0.05	0.97	0.88	*	0.94	0.84	*	0.87	0.76	Yes
13	0.86	0.73	*	1.00	0.92	*	1.00	0.92	*	0.96	0.87	Yes
14	0.83	0.70	*	1.00	0.92	*	1.00	0.92	*	0.95	0.86	Yes
15	0.86	0.73	*	1.00	0.92	*	1.00	0.92	*	0.96	0.87	Yes
16	0.75	0.61	*	1.00	0.92	*	0.97	0.88	*	0.91	0.82	Yes
17	0.78	0.64	*	1.00	0.92	*	1.00	0.92	*	0.93	0.84	Yes
18	0.86	0.73	*	0.97	0.88	*	1.00	0.92	*	0.95	0.85	Yes
19	0.86	0.73	*	0.97	0.88	*	1.00	0.92	*	0.95	0.85	Yes
20	0.86	0.73	*	0.97	0.88	*	0.97	0.88	*	0.94	0.83	Yes
21	0.78	0.64	*	1.00	0.92	*	1.00	0.92	*	0.93	0.84	Yes
22	0.83	0.70	*	1.00	0.92	*	1.00	0.92	*	0.95	0.86	Yes
23	0.83	0.70	*	1.00	0.92	*	1.00	0.92	*	0.95	0.86	Yes

Notes: V: Aiken's V; LL: Lower limit; p: p-value; SAV: Summary of Aiken's V; SLL: Summary of lower limit; II: Inclusion of item.

As exhibited in **Table 3**, the means of the seven subdimensions possess similar values for the three criteria: greater than 3.3 in the criterion of clarity, greater than 3.6 in the criterion of appropriateness, and greater than 3.8 in the criterion of relevance. The latter two represent an average score very close to the maximum assessment score possible.

In relation to kurtosis, it is clear that in the criterion of clarity, the majority of values are negative; meanwhile, in the other two criteria, the scores for kurtosis are all positive, which points to a high proximity of these scores to the mean. It is notable that in all criteria and for all subdimensions, the asymmetric coefficients are negative, signifying that the median is greater than the mean, producing a greater concentration of scores to the right of the mean. This indicates the stability of the high scores and the atypical character of the low scores.

Regarding the measures of standard deviation and the respective coefficients of variation, it is evident that for the criteria of appropriateness and relevance, all of the subdimensions show percentages lower than 13%, with the exception of subdimension 1, which presents 16.9% in the appropriateness criterion and 13.2% in the relevance criterion. Also, notable is a 0% coefficient of variation with a mean equal to 4 in subdimensions 4 and 7 for the appropriateness criterion, as well as in subdimensions 5 and 7 for the relevance criterion, meaning that the judges assigned the maximum score, demonstrating the stability of the design of the items in these subdimensions.

Finally, in terms of variability, the criterion that presents the greatest dispersion of scores is clarity. Nevertheless, in this criterion the asymmetry remains negative, and the coefficient of variation does not exceed 24% in any of the subdimensions.

Inferential Analysis of Content Validity Through Expert Judgment

For each item, a summary measure of the Aiken's V coefficient and the lower limit of the confidence interval was calculated for the three criteria considered. The decision on the inclusion of an item was made by combining both indicators (**Table 4**).

As demonstrated in **Table 4**, according to the judges' evaluation, each item passes the inter-judge consistency test when considering the three criteria together, which substantiates the retention of all items. Regarding the p-value, both for the appropriateness criterion and for the relevance criterion, all the items

Table 5. Results of inter-judge consistency by dimension for the sufficiency criterion

Dimension	Sufficiency		
	Aiken's V coefficient	Lower limit	Inclusion of item
D1	0.80	0.67	Yes
D2	0.94	0.84	Yes
D3	0.94	0.84	Yes

Table 6. Kendall's W among 12 expert judgements by dimension

	D1. Teaching preparation (items 1 to 8)	p-value	D2. MWS components and geneses (items 9 to 16)	p-value	D3. Vertical planes (items 17 to 23)	p-value
Clarity	0.0023	< 0.001	0.0079	0.0014	0.0017	< 0.001
Appropriateness	0.0017	< 0.001	0.0001	< 0.001	0.0004	< 0.001
Relevance	0.0023	< 0.001	0.0006	< 0.001	0.0002	< 0.001

Table 7. Adjusted item 7 from the questionnaire

No	Adjusted version of the item
7	In relation to the design of tasks for the teaching of homothecy, Thales' theorem, and similarity, what is your main source of information? a. I use the tasks from the school textbook or the teacher's didactic guide. b. I don't use information sources, I create tasks. c. I use tasks available on websites. d. I use tasks proposed in scientific studies. e. I use other support documents for task design (mathematics books, or others).

show a p-value less than the significance, which allows confirming the conservation of the items. *The average Aiken's V coefficient corresponds to 0.92,* considering all 23 items, which indicates a high level of internal consistency. In the analysis of the individual criterion, clarity stands out as the weakest, with ten items identified as requiring adjustments in their wording (see column 2 and column 3 of **Table 4**), specifically, items 1, 2, 7, 9, 10, 11, 12, 16, 17, and 21. However, in the analysis of the p-value for the clarity criterion, only item 2 has a value higher than the level of significance.

Meanwhile, the criterion of sufficiency was analyzed by dimension, as shown in **Table 5**. The results validate the sufficiency of the items by dimension.

To analyze the concordance among the judgements of the 12 experts, Kendall's W coefficient was applied by subdimension (see **Table 6**).

As demonstrated in **Table 6**, while the values of Kendall's W coefficient are low, the p-value provides sufficient evidence to reject the null hypothesis in favor of the alternative hypothesis with a significance level of 0.05; that is, a significant degree of agreement exists among the judges.

Adjustments to the Questionnaire

The majority of the expert judges' observations regarding the items that needed revision were related to expanding some of the questions to be open-ended or including an option for the participants to specify or provide details regarding other possible answers.

As indicated previously, ten items have been identified that require adjustments according to the criterion of clarity in the question wording and/or their answer choices. Of these items, 7 and 11 are shown below, as they represent the most significant changes in terms of the purpose or theoretical nature of the item.

Item 7–corresponding to dimension (1) teaching preparation and subdimension (S2), which refers to mathematical skills in the school curriculum–was modified in two aspects. One change was semantic, in an option (b), in which a word was eliminated from the phrase that initially read "I create unpublished tasks." The other change was related to wording of the options, since the item concerns the main source of information for the design of tasks; this involved fully eliminating one option and replacing it with another, along with making the final option more precise. The adjusted item is shown in **Table 7**.

Meanwhile, Item 11–related to dimension (2) MWS components and geneses and subdimension (3), which refers to the components of the MWS–was modified in terms of the theoretical nature of the item. This change

Table 8. Adjusted item 11 from the questionnaire

No	Adjusted version of the item
11	Which types of validation arguments do you prioritize in the teaching of geometry?
	a. Formal demonstrations of theorems.
	b. Those centered on students' practical experience.
	c. Conjectures, explanations, or justifications that arise from the students.
	d. Validations that combine aspects of practical experience and theory.
	e. I generally do not use activities that involve validation arguments in geometry teaching).

followed one of the evaluators suggesting revision to avoid confusion between the notions of *proof* and *demonstration*. Additionally, the item was improved to attend to the balance between the theoretical lexicon of the MWS and the teachers' understanding of the terms, using the national secondary school curriculum as a reference (Mineduc, 2015) to guarantee the use of a common theoretical vocabulary that reflects teachers' work more closely. The adjusted version of this item is exhibited in **Table 8**.

The modifications were made with attention to the balance between the elements of the theoretical framework and teachers' understanding. In this manner, while it is important to ensure that the design of the questions adequately addresses the theoretical principles of the MWS according to the subdimensions proposed, it is also true that the use of a theoretical lexicon, with specific terms such as *proof*, *visualization*, *discursive processes*, *artifacts*, *tools*, or others could affect or distort teachers' interpretations. Therefore, these terms were adjusted based on more technical vocabulary, using the secondary school mathematics curriculum as a frame of reference.

CONCLUSION

This article has described the process of design and content validation of the MWS-IG questionnaire, which explores the characteristics of secondary education teachers' mathematical work in geometry teaching, specifically in the topics of homothecy, similarity, and Thales' theorem. The instrument is theoretically underpinned by the MWS, and it is structured based on the vertical planes, components, and geneses of the MWS (Kuzniak, 2022), also considering aspects associated with teaching preparation. The proposed aim of this study responds to the scarcity of research on the development of valid instruments to study how mathematics teachers organize their geometry teaching practice.

The process of design and validation of the instrument was organized in four stages. First, the instrument was constructed as a brief and self-reported forced-choice questionnaire, in which the respondent is expected to choose only one of five possible options. The content validation was undertaken using Aiken's V coefficient the Kendall's W coefficient.

For evaluation by dimension, the Kendall's W coefficient test was utilized, mainly analyzing the p-value with respect to the level of significance previously selected (Legendre, 2005), which showed a significant degree of agreement among the judges for each dimension. The results validate the structure of the dimensions such that it was not necessary to rearrange the items. It is important to highlight that the low values of the W coefficient were due to the scale used (Likert) allowing the same weight to be assigned to distinct items, contrary to other comparative ordinal scales that require the evaluator to order the items based on importance (i.e., ranking).

Regarding the analysis by item, a high level of internal consistency was demonstrated, with an average Aiken's V coefficient of 0.92. The use of this coefficient was not only due to its advantages in terms of interpretation and calculation, but also its background of successful use to validate specialized instruments in the scope of mathematics education, such as a test to measure primary school students' levels of mathematical competence (Zenteno Ruiz et al., 2020) and an observation guide for classroom practices of preschool teachers (Olfos Ayarza et al., 2022). In this case, the individual results per item, collectively considering the criteria of *clarity*, *appropriateness*, and *relevance*, support the retention of the 23 items in question. However, considering only the criterion of *clarity*, ten items were identified with an Aiken's V coefficient of less than 0.8, which necessitated adjustments in their wording and their usage of technical vocabulary in accordance with the qualitative assessments of the expert judges. Based on these evaluations, modifications were made in the wording of the questions or the multiple-choice options on a case-by-case basis.

The majority of the expert judges' observations regarding the items contravened the objective of constructing a forced-response, rapid application instrument oriented toward facilitating the gathering of data for subsequent construct analysis. To remedy this issue, the wording of the items in question was revised to emphasize that the answer choice should be based on what is most prioritized or favored in the teaching process. In short, only those suggestions which did not imply changes to the structure of the item response type were directly attended to.

Ultimately, the present study is focused on the process of design, content validation, and corresponding adjustments of the MWS-IG questionnaire, which is proposed as a contribution to the understanding of teachers' practice based on well-defined dimensions and sustained by the MWS theory, which has been widely utilized to study teachers' work (Panqueban et al., 2024). As part of the larger project within which this work has taken place, piloting processes for the reliability and construct validity of the instrument are planned. In this sense, this instrument possesses multiple functionalities, which are presented here as projections:

(1) its use will allow for the obtention of empirical evidence on a greater scale in relation to geometry teaching, facilitating, for example, quantitative studies for the longitudinal and cross-sectional comparison of groups,

(2) from the information obtained, decisions can be made at the curricular level for teacher education or for the design of proposals for pedagogical improvements,

(3) eventual uses in future studies that consider geometry teaching with emphasis on the *idoine* MWS of the in-service teacher, and

(4) the instrument offers the possibility of being utilized as a theoretical tool to study the variation in what the teacher perceives compared to what they effectively carry out in the classroom.

Author contributions: Both authors have sufficiently contributed to the study and approved the final version of the article.

Funding: This article was funded by National Research and Development Agency of Chile under the Fondo Nacional de Desarrollo Científico y Tecnológico de iniciación 2023, Folio 11230523, whose main researcher is the first author.

Ethics declaration: This study was approved by the Ethics Committee at Universidad Católica del Maule on 19 May 2023. Participants (expert judges) gave their consent to answer the validation questionnaire and the study did not involve confidential personal data.

Declaration of interest: The authors declared no competing interest.

Data availability: Data generated or analyzed during this study are available from the authors on request.

REFERENCES

Aiken, L. R. (1985). Three coefficients for analyzing the reliability and validity of ratings. *Educational and Psychological Measurement, 45*(1), 131–142. https://doi.org/10.1177/0013164485451012

Arteaga-Martínez, B., Macías-Sánchez, J., Pla-Castells, M., & Ramírez-García, M. (2021). Diseño y validación de un instrumento para observación de clases de matemáticas en educación secundaria: Grupo nominal y método Delphi [Design and validation of an instrument for observing mathematics classes in secondary education: Nominal group and Delphi method]. *Revista Electrónica de Investigación y Evaluación Educativa, 27*(2). https://doi.org/10.30827/relieve.v27i2.21812

Arzarello, F., Olivero, F., Paola, D., & Robutti, O. (2002). A cognitive analysis of dragging practises in Cabri environments. *Zentralblatt für Didaktik der Mathematik, 34*(3), 66–72. https://doi.org/10.1007/bf02655708

Arzarello, F., Olivero, F., Paola, D., & Robutti, O. (2007). The transition to formal proof in geometry. In *Theorems in school* (pp. 305–323). Brill. https://doi.org/10.1163/9789087901691_018

Avcu, R. (2022). Pre-service middle school mathematics teachers' personal concept definitions of special quadrilaterals. *Mathematics Education Research Journal, 35*(4), 743–788. https://doi.org/10.1007/s13394-022-00412-2

Ayvaz, Ü., Gündüz, N., & Bozkuş, F. (2017). Understanding of prospective mathematics teachers of the concept of diagonal. *Journal on Mathematics Education, 8*(2), 165–184. https://doi.org/10.22342/jme.8.2.4102.165-184

Chan, E. K. H. (2014). Standards and guidelines for validation practices: Development and evaluation of measurement instruments. In B. Zumbo, & E. Chan (Eds.), *Validity and validation in social, behavioral, and health sciences. Social indicators research series* (vol 54, pp. 9–24). Springer. https://doi.org/10.1007/978-3-319-07794-9_2

Chuaqui, M., & Riera, G. (2011). *Transformaciones en geometría Euclidiana y no Euclidiana* [Transformations in Euclidean and non-Euclidean geometry]. Universidad de Chile.

Clemente, F., & Llinares, S. (2015). Formas del discurso y razonamiento configural de estudiantes para maestros en la resolución de problemas de geometría [Forms of discourse and configural reasoning of students for teachers in solving geometry problems]. *Enseñanza de las Ciencias, 33*(1), 9–27. https://doi.org/10.5565/rev/ensciencias.1332

Costa, V. A., & Del Río, L. S. (2019). Aportes de la geometría dinámica al estudio de la noción de función a partir de un problema geométrico: Un análisis praxeológico [Contributions of dynamic geometry to the study of the notion of function from a geometric problem: A praxeological analysis]. *Bolema, 33*(63), 67–87. https://doi.org/10.1590/1980-4415v33n63a04

CPEIP. (2021). Estándares pedagógicos y disciplinarios para carreras de pedagogía en matemática [Pedagogical and disciplinary standards for mathematics teaching courses]. *Center for Improvement, Experimentation and Pedagogical Research*. https://estandaresdocentes.mineduc.cl/wp-content/uploads/2023/05/Matematica-Media.pdf

Creager, M. A. (2022). Geometric refutations of prospective secondary mathematics teachers. *International Journal of Education in Mathematics, Science and Technology, 10*(1), 74–99. https://doi.org/10.46328/ijemst.1594

Dede, Y. (2010). Mathematics education values questionnaire for Turkish preservice mathematics teachers: Design, validation, and results. *International Journal of Science and Mathematics Education, 9*(3), 603–626. https://doi.org/10.1007/s10763-010-9213-8

Díaz, V. (2022). Design and validation of a test for the types of mathematical problems associated with reading *Comprehension. Education Sciences, 12*(11), Article 795. https://doi.org/10.3390/educsci12110795

Duval, R. (2005). Les conditions cognitives de l'apprentissage de la géométrie: Développement de la visualisation, différenciation des raisonnements et coordination de leur fonctionnements [The cognitive conditions for learning geometry: Development of visualization, differentiation of reasoning and coordination of their functioning]. *Annales de Didactique et de Sciences Cognitives, 10*, 5–53.

Escobar-Pérez, J., & Cuervo-Martínez, Á. (2008). Validez de contenido y juicio de expertos: Una aproximación a su utilización [Content validity and expert judgment: An approach to its use]. *Avances en Medición, 6*(1), 27–36.

Espinoza Salfate, L., Guerrero, G., Barbé Farré, J., & Márquez Salinas, F. (2023). Design and validation of a classroom observation instrument to evaluate the quality of mathematical activity from a gender perspective. *Education Sciences, 13*(3), 266. https://doi.org/10.3390/educsci13030266

Fernández Blanco, T., Godino, J. D., & Pegito, J. A. C. (2012). Razonamiento geométrico y visualización espacial desde el punto de vista ontosemiótico [Geometric reasoning and spatial visualization from the ontosemiotic point of view]. *Bolema, 26*(42a), 39–64. https://doi.org/10.1590/s0103-636x2012000100004

García López, M. M., Romero Albaladejo, I. M., & Gil Cuadra, F. (2021). Efectos de trabajar con GeoGebra en el aula en la relación afecto-cognición [Effects of working with GeoGebra in the classroom on the affect-cognition relationship]. *Enseñanza de las Ciencias, 39*(3), 177–198. https://doi.org/10.5565/rev/ensciencias.3299

Gómez-Chacón, I. M., Kuzniak, A., & Vivier, L. (2016). El rol del profesor desde la perspectiva de los Espacios de trabajo matemático [The role of the teacher from the perspective of mathematical workspaces]. *Bolema, 30*(54), 1–22. https://doi.org/10.1590/1980-4415v30n54a01

Guzmán Retamal, I., Pino-Fan, L. R., & Arredondo, E. H. (2020). Paradojas didácticas observadas en la gestión de los teoremas de Euclides [Didactic paradoxes observed in the management of Euclid's theorems]. *Bolema, 34*(67), 651–677. https://doi.org/10.1590/1980-4415v34n67a15

Hankeln, C., & Greefrath, G. (2021). Mathematische modellierungskompetenz fördern durch lösungsplan oder dynamische geometrie-software? Empirische ergebnisse aus dem LIMo-projekt [Promote mathematical modeling skills through solution plans or dynamic geometry software? Empirical results from the LIMo project]. *Journal für Mathematik-Didaktik, 42*(2), 367–394. https://doi.org/10.1007/s13138-020-00178-9

Henríquez-Rivas, C. (2017). El espacio de trabajo matemático como una herramienta para el estudio de la práctica en el aula del profesor [The mathematical working space as a tool for the study of the teacher's classroom practice]. In I. M. Gómez-Chacón et al. (Eds.), *Proceedings Fifth MWS Symposium* (pp. 105–116). University of Western Macedonia.

Henríquez-Rivas, C., & Kuzniak, A. (2021). Profundización en el trabajo geométrico de futuros profesores en entornos tecnológicos y de lápiz y papel [Deepening the geometric work of future teachers in technological environments and pencil and paper]. *Bolema, 35*(71), 1550–1572. https://doi.org/10.1590/1980-4415v35n71a15

Henríquez-Rivas, C., Kuzniak, A., & Masselin, B. (2022). The idoine or suitable MWS as an essential transitional stage between personal and reference mathematical work. In A. Kuzniak, E. Montoya-Delgadillo, & P. R. Richard (Eds.), *Mathematical work in educational context. The Perspective of the Theory of Mathematical Working Spaces* (pp. 121–146). Springer. https://doi.org/10.1007/978-3-030-90850-8_6

Henríquez-Rivas, C., Ponce, R., Carillo Yáñez, J., Climent, N., & Espinoza-Vásquez, G. (2021). Trabajo matemático de un profesor basado en tareas y ejemplos propuestos para la enseñanza [Mathematical work of a teacher based on tasks and examples proposed for teaching]. *Enseñanza de las Ciencias, 39*(2), 123–142. https://doi.org/10.5565/rev/ensciencias.3210

Herbst, P., Cheah, U. H., Richard, P. R., & Keith, J. (2018). *International perspectives on the teaching and learning of geometry in secondary schools*. Springer. https://doi.org/10.1007/978-3-319-77476-3

Hershkowitz, R. (1990). Psychological aspects of learning geometry. In P. Nesher, & J. Kilpatrick (Eds.), *Mathematics and cognition* (pp. 70–95). Cambridge University Press. https://doi.org/10.1017/CBO9781139013499.006

Jones, K., & Tzekaki, M. (2016). Research on the teaching and learning of geometry. In G. Gutiérrez, C. Leder, & P. Boero (Eds.), *The second handbook of research on the psychology of mathematics education: The journey continues* (pp. 109–149). Sense Publishers. https://doi.org/10.1007/978-94-6300-561-6_4

Kirkland, P., Cheng, Y., & McNeil, N. (2024). A validity argument for a brief assessment of mature number sense. *Journal for Research in Mathematics Education, 55*(1), 51–67. https://doi.org/10.5951/jresematheduc-2022-0071

Kuzniak, A. (2006). Paradigmes et espaces de travail géométriques. Éléments d'un cadre théorique pour l'enseignement et la formation des enseignants en géométrie [Geometric paradigms and workspaces. Elements of a theoretical framework for teaching and teacher training in geometry]. *Canadian Journal of Science, Mathematics and Technology Education, 6*, 167–187. https://doi.org/10.1080/14926150609556694

Kuzniak, A. (2011). L'espace de travail mathématique et ses genèses [The mathematical workspace and its geneses]. *Annales de Didactique et de Sciences Cognitives, 16*, 9–24.

Kuzniak, A. (2018). Thinking about the teaching of geometry through the lens of the theory of geometric working spaces. In P. Herbst, U. H. Cheah, P. R. Richard, & K. Jones (Eds.), *International perspectives on the teaching and learning of geometry in secondary schools* (pp. 5–21). Springer. https://doi.org/10.1007/978-3-319-77476-3_2

Kuzniak, A. (2022). The theory of mathematical working spaces–Theoretical characteristics. In A. Kuzniak, E. Montoya-Delgadillo, & P. R. Richard (Eds.), *Mathematical work in educational context: The perspective of the theory of mathematical working spaces* (pp. 3–31). Springer. https://doi.org/10.1007/978-3-030-90850-8_1

Kuzniak, A., & Nechache, A. (2021). On forms of geometric work: A study with pre-service teachers based on the theory of mathematical working spaces. *Educational Studies in Mathematics, 106*(2), 271–289. https://doi.org/10.1007/s10649-020-10011-2

Kuzniak, A., & Richard, P. (2014). Espacios de trabajo matemático. Puntos de vista y perspectivas [Mathematical workspaces. Points of view and perspectives]. *Revista Latinoamericana de Investigación en Matemática Educativa, 17*(4), 5–15. https://doi.org/10.12802/relime.13.1741a

Kuzniak, A., Montoya Delgadillo, E., & Richard, P. (2022). *Mathematical work in educational context: The perspective of the theory of mathematical working spaces*. Springer. https://doi.org/10.1007/978-3-030-90850-8

Kuzniak, A., Tanguay, D., & Elia, I. (2016). Mathematical working spaces in schooling: An introduction. *ZDM–Mathematics Education, 48*, 721–737. https://doi.org/10.1007/s11858-016-0812-x

Lagrange, J. B., & Richard, P. (2022). Instrumental genesis in the theory of MWS: Insight from didactic research on digital artifacts. In A. Kuzniak, E. Montoya-Delgadillo, & P. R. Richard (Eds.), *Mathematical work in educational context: The perspective of the theory of mathematical working spaces* (pp. 211–228). Springer. https://doi.org/10.1007/978-3-030-90850-8_9

Lederman, R. P. (1993). Comparative and consistent approaches to exploratory research. *The American Journal of Maternal/Child Nursing, 18*(2), Article 107. https://doi.org/10.1097/00005721-199303000-00009

Legendre, P. (2005). Species associations: The Kendall coefficient of concordance revisited. *Journal of Agricultural, Biological, and Environmental Statistics, 10*(2), 226–245. https://doi.org/10.1198/108571105x46642

Machuca Pérez, K., & Montoya Delgadillo, E. (2022). The mathematical work of Chilean secondary teachers in the domain of probability. *Canadian Journal of Science, Mathematics and Technology Education, 22*, 558–575. https://doi.org/10.1007/s42330-022-00235-0

Magaña Medina, D. E., Hernández-Mena, V., Aguilar Morales, N., & Sánchez Escobedo, P. A. (2023). Apoyo de pares y expectativas de resultado en STEM: Desarrollo y validación de un instrumento [Peer support and outcome expectations in STEM: Development and validation of an instrument]. *Revista Electrónica de Investigación Educativa, 25*, 1–12. https://doi.org/10.24320/redie.2023.25.e06.4274

Martínez-Mora, E., & Camargo-Uribe, L. (2021). Construcción de triángulos, recurso para impulsar un ambiente de indagación [Building triangles, a resource to promote an environment of inquiry]. *Revista Internacional de Investigación en Educación, 14*, 1–24. https://doi.org/10.11144/javeriana.m14.ctri

Martin-Romera, A., & Molina Ruiz, E. (2017). Valor del conocimiento pedagógico para la docencia en educación secundaria: Diseño y validación de un cuestionario [Value of pedagogical knowledge for teaching in secondary education: Design and validation of a questionnaire]. *Estudios Pedagógicos, 43*(2), 195–220. https://doi.org/10.4067/s0718-07052017000200011

Mineduc. (2015). Bases curriculares 7º básico a 2º medio [Curricular bases 7th basic to 2nd secondary]. *Ministry of Education of Chile*. https://media.mineduc.cl/wp-content/uploads/sites/28/2017/07/Bases-Curriculares-7º-básico-a-2º-medio.pdf

Mohr-Schroeder, M., Ronau, R., Peters, S., Lee, C., & Bush, W. (2017). Predicting student achievement using measures of teachers' knowledge for teaching geometry. *Journal for Research in Mathematics Education, 48*(5), 520–566. https://doi.org/10.5951/jresematheduc.48.5.0520

Montoya-Delgadillo, E., & Vivier, L. (2016). Mathematical working space and paradigms as an analysis tool for the teaching and learning of analysis. *ZDM–Mathematics Education, 48*(6), 739–754. https://doi.org/10.1007/s11858-016-0777-9

Nechache, A., & Gómez-Chacón, I.M. (2022). Methodological aspects in the theory of mathematical working spaces. In A. Kuzniak, E. Montoya-Delgadillo, & P. R. Richard (Eds.), *Mathematical work in educational context: The perspective of the theory of mathematical working spaces* (pp. 33–56). Springer. https://doi.org/10.1007/978-3-030-90850-8_2

Olfos Ayarza, R., Goldrine Godoy, T., Morales Candia, S., Estrella, S., Morales Ibarra, G., Pinto Wong, A., & Reyes Santander, P. (2022). Pauta para evaluar la dimensión práctica de la capacidad de enseñanza de la matemática en maestras de infantil en formación [Guidelines for assessing the practical dimension of the ability to teach mathematics in pre-school teachers]. *Revista de Investigación en Didáctica de la Matemática, 16*(4), 343–364. https://doi.org/10.30827/pna.v16i4.22125

Panqueban, D., Henríquez-Rivas, C., & Kuzniak, A. (2024). Advances and trends in research on mathematical working spaces: A systematic review. *Eurasia Journal of Mathematics, Science and Technology Education*, *20*(6), Article em2450. https://doi.org/10.29333/ejmste/14588

Penfield, R. D., & Giacobbi Jr., P. R. (2004). Applying a score confidence interval to Aiken's item content-relevance index. *Measurement in Physical Education and Exercise Science, 8*(4), 213–225. https://doi.org/10.1207/s15327841mpee0804_3

Pincheira-Hauck, N., & Vásquez-Ortiz, C. (2018). Conocimiento didáctico-matemático para la enseñanza de la matemática elemental en futuros profesores de educación básica: Diseño, construcción y validación de un instrumento de evaluación [Didactic-mathematical knowledge for teaching elementary mathematics in future teachers of basic education: Design, construction and validation of an evaluation instrument]. *Estudios Pedagógicos, 44*(1), 25–48. https://doi.org/10.4067/s0718-07052018000100025

Prior, J. M., & Torregrosa, G. G. (2020). Razonamiento configural y espacio de trabajo geométrico en la resolución de problemas de probar [Configural reasoning and geometric workspace in solving testing problems]. *Bolema, 34*(66), 178–198. https://doi.org/10.1590/1980-4415v34n66a09

Radford, L. (2017). On inferentialism. *Mathematics Education Research Journal, 29*(4), 493–508. https://doi.org/10.1007/s13394-017-0225-3

Richard, P., Venant, F., & Gagnon, M. (2019). Issues and challenges in instrumental proof. In G. Hanna, D. A. Reid, & M. de Villiers (Eds.), *Proof technology in mathematics research and teaching: Mathematics education in the digital era, vol. 14* (pp. 139–172). Springer. https://doi.org/10.1007/978-3-030-28483-1_7

Seguí, J. F., & Alsina, Á. (2023). Evaluando el conocimiento especializado para enseñar estadística y probabilidad: Elaboración y validación del cuestionario MTSK-estocástico [Assessing specialized knowledge for teaching statistics and probability: Development and validation of the MTSK-stochastic questionnaire]. *Uniciencia, 37*(1), 84–105. https://doi.org/10.15359/ru.37-1.5

Sinclair, N., Bartolini Bussi, M., de Villiers, M., Jones, K., Kortenkamp, U., Leung, A., & Owens, K. (2017). Geometry education, including the use of new technologies: A survey of recent research. In G. Kaiser (Eds.), *Proceedings of the 13th International Congress on Mathematical Education* (pp. 277–287). Springer. https://doi.org/10.1007/978-3-319-62597-3_18

Skjong, R., & Wentworth, B. (2001). Expert judgement and risk perception. In *Proceedings of the International Offshore and Polar Engineering Conference* (pp. 537–544).

Sunzuma, G., & Maharaj, A. (2020). In-service secondary teachers' teaching approaches and views towards integrating ethnomathematics approaches into geometry teaching. *Bolema, 34*(66), 22–39. https://doi.org/10.1590/1980-4415v34n66a02

Tachie, S. A. (2020). The challenges of South African teachers in teaching Euclidean geometry. *International Journal of Learning, Teaching and Educational Research, 19*(8), 297–312. https://doi.org/10.26803/ijlter.19.8.16

Torregrosa, G., & Quesada, H. (2007). Coordinación de procesos cognitivos en geometría [Coordination of cognitive processes in geometry 9. *Revista latinoamericana de Investigación en Matemática Educativa, 10*(2), 275–300.

Vásquez Ortiz, C. A., Alsina Pastells, Á., Pincheira Hauck, N. G., Gea Serrano, M. M., & Chandia Muñoz, E. (2020). Construcción y validación de un instrumento de observación de clases de probabilidad [Construction and validation of a probability class observation instrument]. *Enseñanza de las Ciencias, 38*(2), 25-43. https://doi.org/10.5565/rev/ensciencias.2820

Verdugo-Castro, S., Sánchez-Gómez, M. C., & García-Holgado, A. (2022). Opiniones y percepciones sobre los estudios superiores STEM: Un estudio de caso exploratorio en España [Opinions and perceptions about STEM higher education: An exploratory case study in Spain]. *Education in the Knowledge Society,* 23. https://doi.org/10.14201/eks.27529

Zakaryan, D., & Sosa, L. (2021). Conocimiento del profesor de secundaria de la práctica matemática en clases de geometría [High school teacher's knowledge of mathematical practice in geometry classes]. *Educación Matemática, 33*(1), 71–97. https://doi.org/10.24844/em3301.03

Zenteno Ruiz, F. A., Carhuachín Marcelo, A. I., & Rivera Espinoza, T. A. (2020). Uso de software educativo interactivo para la enseñanza y aprendizaje de la matemática en educación básica, Región Pasco [Use of interactive educational software for teaching and learning mathematics in basic education, Pasco Region]. *Horizonte de la Ciencia, 10*(19), 178–190. https://doi.org/10.26490/uncp.horizonteciencia.2020.19.596

APPENDIX A: FINAL VERSION OF THE MWS-IG QUESTIONNAIRE

Table A1. Final version of the MWS-IG questionnaire

Dimension 1: Teaching preparation	
(S1) Teaching organization. Referring to general aspects related to planning for geometry teaching, aspects of the implementation of teaching and evaluation, particularly focused on specific topics.	
Item 1	In your lesson planning, how many weeks do you devote to the preparation of teaching homothecy, Thales' theorem, and similarity?
Item 2	In terms of the implementation of the topics of homothecy, Thales' theorem, and similarity, how to you organize these topics in your teaching?
Item 3	How is the learning of the topics of homothecy, Thales' theorem, and similarity evaluated?
(S2) School curriculum. Referring to the *mathematical skills* of the school curriculum that are prioritized in the teaching of specific geometric topics and the *task design* that occurs in relation to said skills. These mathematical skills include representing, problem-solving, modeling, and argumentation and communication	
Item 4	Which mathematical skill do you prioritize in the teaching of homothecy?
Item 5	Which mathematical skill do you prioritize in the teaching of Thales' theorem?
Item 6	Which mathematical skill do you prioritize in the teaching of similarity?
Item 7	In relation to the design of tasks for the teaching of homothecy, Thales' theorem, and similarity, what is your main source of information?
Item 8	In relation to approaches for the learning proposed in the school curriculum, what aspects do you consider for the teaching of homothecy?
Dimension 2: MWS components and geneses	
(S3) Components. Referring to the *components* of the MWS related to its epistemological and cognitive planes activated in the teaching of geometry.	
Item 9	Which visualization activities do you prioritize in the teaching of geometry?
Item 10	Which types of constructions do you prioritize in the teaching of geometry?
Item 11	Which types of validation arguments do you prioritize in the teaching of geometry?
Item 12	What role of geometric objects do you prioritize in your class?
Item 13	Which theoretical aspects do you prioritize when addressing the geometric topics that you teach?
Item 14	Which support tools do you tend to utilize in geometry teaching?
(S4) Geneses. Referring to the *geneses* of the MWS that allow connection between components of the epistemological and cognitive planes activated in the teaching of specific geometric topics (similarity, homothecy, and Thales' theorem).	
Item 15	Which *representations* do you prioritize when teaching the topics of similarity, homothecy, and Thales' theorem?
Item 16	What types of tasks or activities do your encourage the use of *instruments* for when teaching the topics of similarity, homothecy, and Thales' theorem?
Item 17	For what purposes do you encourage argumentation or validation when teaching the topics of similarity, homothecy, and Thales' theorem?
Dimension 3: MWS vertical planes	
(S5) [Sem-Ins] Vertical plane. Referring to the *[Sem-Ins] vertical plane*, which is associated with semiotic and instrumental geneses, when artefacts are used for construction under certain conditions, for exploring representations, or for discovering new properties, without the purpose of validation.	
Item 18	Which of the following statements best represents your practice when teaching Thales's theorem?
Item 19	Which of the following statements best represents your practice when teaching homothecy?
(S6) [Ins-Dis] Vertical plane. Referring to the [Ins-Dis] vertical plane, associated with instrumental and discursive geneses, when the proof is based on experimentation or exploration and employs and artefact, or on the justification of a construction.	
Item 20	Which of the following statements best represents your teaching practice in relation to the properties of homothecy?
Item 21	Which of the following statements best represents your teaching practice in relation to the validation of Thales' theorem?
(S7) [Sem-Dis] Vertical plane. Referring to the *[Sem-Dis]* vertical plane, which is associated with semiotic and discursive geneses, when a proof is coordinated with the process of visualization of objects represented and the status of the reasoning involved may vary.	
Item 22	Which of the following statements best represents your teaching practice in relation to homothecy?
Item 23	Which of the following statements best represents your teaching practice in relation to Thales' theorem?