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Estimation of uncertainty for problem solving by fuzzy mathematics tools

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Abstract: The occurrence of uncertainty should be taken into consideration at all stages of solving research problems, from assignment to evaluation. The aim of this paper is to show possibilities of processing and evaluation of experiment based on the analysis of uncertainty using fuzzy math. We will mainly focus on the cases where it is impossible to evaluate results of experiment using standard statistical methods, due to the small extent of the investigation file. For example, this situation occurs very often with specialized training, not only in technical colleges. The paper will also feature a sample of an evaluation experiment concerning the extent of realization of chosen teaching methods of numerical math, which took place in the previous academic year at the Transport Faculty of the University of Pardubice.

Keywords: uncertainty, algorithm, evaluation, experiment, fuzzy, applied mathematics, Matlab.

Introduction

Measurements under uncertainty and vagueness have their limitations that are given, among other things, by the absence of a prototype unit of the measured quality. In this case, it is necessary to introduce "prototypes" of qualities which are then used for comparing the qualities of the measured sample. This article deals with an indication of the possibility of estimating uncertainty such "measurements" especially when we cannot use classical statistical methods due to the small scale of the primary data file.

Theoretical Background

Determination of knowledge is an example of a measurement that is fraught with great uncertainty. It is usually an objectified valid questionnaire (test) that serves here as a measuring means. It is divided into individual sets of semantically related items. The respondent reacts to the items with a response that may be scorable either by dichotomy (0-1, or possibly Yes-No, etc.) or by using a multi-level scale (0-1-2-3-4 with detailed interpretations, etc.). The uncertainty in estimation of knowledge then originates both in the choice of questionnaire items (the author of the questionnaire chooses from a set of options for the form and content of the item) and also in the response scoring method (there are various options for choosing the assessment scale and its interpretation). The overall uncertainty in the determination of the level of knowledge is therefore affected by the uncertainty that originates in the questionnaire (the respondent cannot affect it) and the uncertainty arising from the uncertainty of scoring as well as of the form of reactions recorded as affected by the level of knowledge of the respondent. In our case, we assume the measurement of knowledge for simplification that the uncertainties associated with the questionnaire and scoring thereof are constant in time and with regard to the respondents. The degree of the respondent's knowledge is assessed from a knowledge function determined from the respective degree of uncertainty for the j-th item of the questionnaire. In our case, we shall use the relative frequency for expert estimation of comparison with the ideal state of knowledge.

Estimating the knowledge from measurement uncertainty.
We shall follow a category - the ability to think abstractly that we shall mark as A, characterized directly by the traceable system k of phenomena $j_1, j_2, \cdots, j_k$. This system shall be examined with a set of tests. Let μ_i be success rates of an individual in the i-th set, $i = 1, 2, \cdots, k$. Then the fuzzy set $A = \{j_1/\mu_1;\ j_2/\mu_2; \cdots; j_k/\mu_k\}$, where $\mu_A(j_i) = \mu_i$, represents an estimate of the category level A of the individual concerned. The value of the μ function may be, for example, the relative number of the successfully solved items of the relevant set or a relatively weighted score with the informational degree of difficulty or an expert estimate.

The difference of two individuals in the A category levels can be measured using the generalized Hamming distance relationship: $d(A_1; A_2) = \sum_{i=1}^{k} \left|\mu_{A_1(j_i)} - \mu_{A_2(j_i)}\right|$ or relatively: $\delta(A_1; A_2) = d(A_1; A_2)/k\,; \delta(A_1; A_2) \in \langle 0; 1\rangle$, where A_1 or A_2 respectively are estimates of the A category with the first, or the second individual, respectively. Another way to measure differences is by using the generalized quadratic Euclidean norm: $e(A_1; A_2) = \sqrt{\sum_{i=1}^{k}\left(\mu_{A_1(j_i)} - \mu_{A_2(j_i)}\right)^2}$ or relatively: $\varepsilon(A_1; A_2) = e(A_1; A_2)/\sqrt{k};\ \varepsilon(A_1; A_2) \in \langle 0; 1\rangle$. The difference is the greater, the more $\delta(A_1; A_2)$ or $e(A_1; A_2)$ respectively approaches one.

Furthermore, for the fuzzy set A we shall define the fuzzy set $\overline{A}$ so that: $\overline{A} = \{j_1/\vartheta_1;\ j_2/\vartheta_2; \cdots; j_k/\vartheta_k\}$, where $\vartheta_i = 0$, if $\mu_i < 0{,}5$ and $\vartheta_i = 1$, if $\mu_i \geq 0{,}5$, $i = 1,2, \cdots, k$. The fuzzy set $\overline{A}$ shall be referred to as the kernel of the fuzzy set A. The kernel $\overline{A}$ can be unambiguously assigned with the classical set, which consists of the elements x_i, for which $\vartheta_i = 1$. Using the estimates A and $\overline{A}$ we define two fuzziness indexes category of the level A: linear fuzziness index $\vartheta(A)$ by the relation $\vartheta(A) = 2/k \cdot d(A_1; A_2); 0 \leq \vartheta(A) \leq 1$, and quadratic fuzziness index $\eta(A)$ by the relation $\eta(A) = 2/\sqrt{k} \cdot e(A_1; A_2); 0 \leq \eta(A) \leq 1$.

Evaluating the vague data.
If the teacher is to assess the level of knowledge of the student, they first compare the student with the "ideal state" of knowledge. "The ideal student" is a standardized formulation of the basic factors of consciousness, through which we can estimate the intensity of determinants on a defined scale (e.g. 1 - 2 - 3 - 4 or excellent - very good - good - failed). The extent of the scale is proportional to the weight of the relevant factors. The values of each factor scales are determined by the teacher according to the instructions for test evaluation and according to their own experience. The values are thus fuzzy.

The resulting data for individual factors can be understood as triangular the fuzzy numbers A, determined using real numbers $a_0, a_1, a_2\ (a_1 < a_0 < a_2)$ so that the membership function $\mu_A: R^+ \to \langle 0; 1\rangle$ of the fuzzy number is:

$$\mu_A(x) = \begin{cases} 0;\ \text{for } x \leq a_1 \\ \dfrac{x - a_1}{a_0 - a_1};\ \text{for } x \in \langle a_1; a_0\rangle \\ \dfrac{a_2 - x}{a_2 - a_0};\ \text{for } x \in \langle a_1; a_2\rangle \\ 0;\ \text{for } x \geq a_2 \end{cases}$$

The numbers a_1 and a_2 here represent the level of uncertainty in the evaluation of the relevant factor by the estimation a_0. The total test score with n items is determined by the sum of the fuzzy numbers $A_1 + A_2 + \cdots + A_n$ and again is the fuzzy number, as determined by real numbers a_0, a_1, a_2: $a_0 = \sum_{i=1}^{n} a_0^i\,;\ a_1 = \sum_{i=1}^{n} a_1^i\,;\ a_2 = \sum_{i=1}^{n} a_2^i$ from the corresponding triplets $\left[a_0^i;\ a_1^i;\ a_2^i\right]$ of the respective fuzzy numbers of the various factors $A_i: i = 1,2, \cdots, n$. The outcome of the investigation can be considered a certain α-section of the fuzzy number A of the total score, α shall be chosen from the interval $(0; 1)$ depending on the requirement of precision, which can then be estimated in percentage as $(1 - \alpha) \cdot 100\%$.

Standardization of estimation of the test response results.
Traditional methods of statistical processing of test results make use of the assumption of equivalence and independence of individual questions from the set. Equivalence and independence issues of the questions are dependent on the respondents, and therefore they cannot be determined in advance through the test structure. In view of the generality, we shall assume that we are working with a test which consists of items that are assigned in the process with numbers $\mu \in \langle 0; 1 \rangle$. When compiling and evaluating the test, we encounter some problems. For example, the respondent focuses on simple, easy items while pushed aside the difficult, yet often more valuable ones. Alternatively, they see success in solving difficult items and leaves aside. The evaluator may attempt to subjectively estimate the difficulty of each item and somehow bonify the harder ones. Here, however, they face problems estimating the difficulty and subsequent modifications of the evaluation. The evaluator must draw conclusions from individual respondents' reactions, taking into account questions of varying difficulty and subjectivity of assigning values to the μ items. The following presented method solves some of these problems in a simple way and is very suitable for computer processing.

Suppose we evaluate individual responses with the number $\mu \in \langle 0; 1 \rangle$, where 0 represents one extreme value of the responses and 1 the other. Values between the extremes represent the degree of position of such evaluation between the two extremes. Then suppose that we have the test results with m items solved by n respondents. For $i = 1,2,\cdots,m; j = 1,2,\cdots,n$ we shall mark μ_{ij} evaluation of $\mu_i = \frac{1}{n} \cdot \sum_{j=1}^{n} \mu_{ij}$ the i-th item solved by the j-th respondent. Let the average degree assigned to the i-th item. Then we can determine the difficulty of the individual items in the form of

$$v_i = \begin{cases} 1 + \frac{1}{2} \cdot [\mu_i \cdot \log_2 \mu_i + (1 - \mu_i) \cdot \log_2 (1 - \mu_i)]; \text{for } \mu_i \in \langle 0; 0{,}5) \\ -\frac{1}{2} \cdot [\mu_i \cdot \log_2 \mu_i + (1 - \mu_i) \cdot \log_2 (1 - \mu_i)]; \text{for } \mu_i \in \langle 0{,}5; 1 \rangle \end{cases}$$

(here, we define $0 \cdot \log_2 0 = 0$).

Next, we shall modify the results depending on the difficulty of each item as follows:

- From the further processing we shall discard those items for which the arithmetic average of the individual responses is equal to zero.
- We shall determine the best and worst result: $K = \max_{\substack{i=1,\cdots,m \\ j=1,\cdots,n}} \mu_{ij};\ k = \min_{\substack{i=1,\cdots,m \\ j=1,\cdots,n}} \mu_{ij}$.
- We shall determine the average degree of μ_i to each item $\mu_i = \frac{1}{n} \cdot \sum_{j=1}^{n} \mu_{ij}$; $i = 1,\cdots,m$.
- For each item, we shall calculate its difficulty v_i; $i = 1,\cdots,m$.
- Individual difficulties are considered weights, which then used for multiplying the individual results: $\mu'_{ij} = \mu_{ij} \cdot v_i; i = 1,\cdots,m; j = 1,\cdots,n$.
- We shall determine the best and the worst result after modification:
  $K' = \max_{\substack{i=1,\cdots,m \\ j=1,\cdots,n}} \mu_{ij};\ k' = \min_{\substack{i=1,\cdots,m \\ j=1,\cdots,n}} \mu_{ij}$.
- We perform linear transformation $\mu''_{ij} = \frac{K-k}{K'-k'} \cdot \left(\mu'_{ij} - K'\right) + K;\ i = 1,\cdots,m; j = 1,\cdots,n$. Provided that $K' > k'$.
- Numbers μ''_{ij} shall be substituted with μ_{ij}.
- The iterative process shall be terminated after the conditions is met $\left|\mu''_{ij} - \mu_{ij}\right| < \varepsilon;\ \varepsilon > 0$ we choose (e.g. $= 10^{-4}$), the process shall be terminated or we proceed to the third point.

Items evaluated by all respondents with the value of 0 or 1 are hardly of any use for the assessment of respondents, they only give some information about the level of questions. The transformation applied causes preserving minima and maxima for each iteration. The purpose of the iteration is a standard estimate of the test solution results, which is based on differences in the difficulties of solving problems in the studied group of respondents. The method converges for the data obtained empirically when the levels of responses are different.

Assume that for the basic linguistic variable - knowledge – we shall interpret the variable μ_i , relating to the i-th item, by its base variable. Then we can express the degree of the expression membership - good knowledge - in the form:

$$\mu_i^z = \frac{1}{2} \cdot \left\{1 + \sin\left[\pi \cdot \left(\mu_i - \frac{1}{2} + \Delta\right)\right]\right\};\ \Delta \in \langle 0; 1\rangle;\ \mu_i \in \langle 0; 1\rangle \qquad (1)$$

In this relationship, the semantics of the expression - good knowledge – is governed by the parameter Δ. With an appropriate choice (e.g. $\Delta = 0; 0{,}25; 0{,}5; 0{,}75; 1$) we can set the importance of this expression to the corresponding linguistic expression of the intuitive level, such as "not too", "sufficiently", "very", etc.

The same base variable μ_i in connection with the performance evaluation in the i-th item shall also be consider for the linguistic variable - success. The degree of membership of the importance of the linguistic expression "poor success rate" may be of the form

$$\mu_i^u = \begin{cases} \left(\mu_i - \frac{1}{2} + \Delta\right)^2 \cdot \frac{1}{(\Delta - 1)^2}; \text{for } \mu_i \leq 1 - \Delta \\ 0; \text{for } 1 - \Delta < \mu_i \leq 1 \end{cases} \qquad (2)$$

where $\Delta \in \langle 0; 1\rangle$. Semantics of this relationship can also be specified with an appropriate choice of the parameter Δ (e.g. $\Delta = 0; 0{,}25; 0{,}5; 0{,}75; 1$) for the respective meanings of "poor success rate", "very poor success rate", "failure". The specification of the semantics allows us to formulate conclusions from our experiment more precisely. The values μ_{ij} obtained by the iterative process allows us to compare more precisely the different approaches to the estimate of the i-th item by the j-th respondent and thus discover the differences among the respondents independently of the influence of chance, which originates in the respondent or in the item.

Evaluation of the Experiment

The pedagogical experiment was carried out last academic year on Jan Perner Transport Faculty in Pardubice. In the first stage, the innovative teaching was conducted in the course Numerical Methods in the first year of the master's degree, and in the second phase, the test was given. In giving the test, the evaluators stressed that they would evaluate not only the numerical accuracy of calculations on a calculator, but also the quality of solutions, which includes the use of images, relevant theories and m-files created in Matlab. Aspects to be checked were both written calculations and scripts stored in the m-files and their outcomes. The test was given to each solver in a pre-printed form and none of the respondents were limited by time, they could work for up to four lessons. The test was given to seven students. The respondents returned their results both in writing and electronically. The solution was evaluated in terms of quality – by the relative frequency (the ratio of the number of correct elementary actions to the number of all the necessary actions in the model solution) and accuracy - in terms of dichotomy (0 if the relative frequency was less than 0.5, and 1 if it was greater than or equal to 0.5 and less than or equal to 1).

The test consisted of four separate tasks. Necessary functions that had been programmed by the students at seminars were stored on the USB drive and were fully available to the respondents.

The first task was focused on solving systems of linear equations by LU decomposition:
Using LU decomposition, find solutions to the following system of equations:

$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ 2 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix}$$

Perform the check by constructing a self-executable script in Matlab, in which you make use of LUR, BackSubst and ForwSubst.

The second problem was to solve the construction of an interpolation polynomial by the Newton's method:
Determine the interpolation polynomial for the function passing through the points $[-2; 0], [-1; 1]$, $[0; 2], [1; 0], [2; 1]$ by the Newton's method. Perform the check by constructing a self-executable script in Matlab, in which you make use of the NewtonPoly function. Plot the situation in Matlab.
The third problem was to solve approximation of the function using the method of least squares:

With the linear polynomial, approximate the function given by the points $[-1;-2],[0;3],[1;7],[2;6]$ using the method of least squares. Perform the check by constructing a self-executable script in Matlab, in which you make use of the LSpoly function. Plot the situation in Matlab.

The fourth problem was related to the issue of solving systems of linear equations by iterative methods:
For the system:

$$\begin{pmatrix} 7 & -4 & 1 \\ 1 & 9 & -3 \\ 4 & 5 & 10 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 33 \\ 13 \\ 45 \end{pmatrix}$$

specify the first three approximations and accuracy by the Gauss-Seidel and Jacobi method. Next, construct a self-executable script in Matlab, in which you make use of the Jacobi and GaussSeidel function that compares the speed of convergence by the methods and determines the number of iterations required to achieve accuracy 10^{-6}. Plot the situation in Matlab using a graph of residues.

In addition to the written solutions, a solution was required for all the problems in the Matlab programme environment. The resulting m-files were saved by the respondents to the appropriate USB disk. The score of individual problems and required elementary actions can be seen in Table 1. In Table 2, there is the score and relative frequencies of the results achieved by the individual respondents.

Table 1. Test Scoring

Example (max pts)	Solution	
	Action	Score
1 (26 pts)	a) LU decomposition	10
	b) X calculation	8
	c) m-file	8
2 (24 pts)	a) table of differences	8
	b) interpolating polynomial	8
	c) m-file + figure	8
3 (23 pts)	a) equation of the line	10
	b) m-file calculation	9
	c) m-file figure	4
4 (27 pts)	a) 3 iterations by the Jacobi method	6
	b) accuracy	2
	c) 3 iterations by the Gauss-Seidel method	6
	d) accuracy	2
	e) m-file of the iteration for the given accuracy	5
	f) m-file figure	6
100	total points	100

Table 2. Results Achieved by the Respondents

Example	Solution							
	Action	Score						
Respondent Number		1	2	3	4	5	6	7
1	a)	6	10	10	10	10	10	10
	b)	4	3	3	8	8	8	4
	c)	8	4	4	0	8	4	4
	Evaluation	0.69	0.65	0.65	0.69	1.00	1.00	0.69
2	a)	4	8	7	8	8	8	8
	b)	2	8	4	8	4	8	8
	c)	7	4	6	4	8	8	3
	Evaluation	0.54	0.83	0.71	0.83	0.83	1.00	0.79
3	a)	0	5	5	10	5	10	5
	b)	9	4	4	0	4	9	4
	c)	4	2	2	0	2	3	2
	Evaluation	0.57	0.48	0.48	0.43	0.48	0.96	0.48
4	a)	3	3	6	6	5	4	6
	b)	2	2	0	2	1	1	1
	c)	1	6	6	6	0	6	6
	d)	0	2	2	2	0	2	1
	e)	5	5	2	0	4	5	3
	f)	6	6	3	0	6	6	3
	Evaluation	0.63	0.89	0.70	0.59	0.59	0.89	0.74
	Overall Evaluation	0.61	0.71	0.64	0.64	0.73	0.96	0.68

Programme in Matlab to estimate the results of the test.
The actual calculation of the fuzzy characteristics of the test is very challenging, especially the linear transformation of the results and their iterations. Therefore, a script was created in Matlab that gradually performs all the necessary calculations are and calculates all of the above characteristics. On the basis of input parameters, the script calculates (A – matrix of the respondents' evaluation, Accuracy - matrix norm: we choose 0,0001 and Maxiter – maximum number of iterations) output parameters (the aforementioned fuzzy characteristics of the matrices A, XA – iterative matrix to the matrix A, B – dichotomous matrix of correctness, XB – iterative matrix to the matrix B). The log of the script exceeds the allowed length of the article, so it will not be featured herein. The readers can obtained it from the author on request.

Characteristics of the test and problem solving by students.
To evaluate the level of the studied group of respondents, we used the average score assigned to results of each problem. These are listed in Table 3.

Table 3. Average Results

Problem	Average Results				Overall Average
	1.	2.	3.	4.	
Quality	0,6971	0,7143	0,1986	0,6343	0,5611
Solution	0,8571	0,8571	0,1429	0,7143	0,6429

Using the data in Table 3 for the overall average, we shall estimate the level of credibility of the following statements:
"The average student has a good quality of knowledge" (relationship (1); $\Delta = 0; \mu = 0{,}5611$) by the value $\mu_z = 0{,}5954$.
"The average student has a good knowledge of the solution" (relationship (1); $\Delta = 0; \mu = 0{,}6429$) by the value $\mu_z = 0{,}717$.

Similarly, using the values for the overall average from Table 3, we shall estimate assuming the semantics of the term "moderate" given by the parameter $\Delta = 0{,}5$ the level of credibility of the statement:
"The average student has a very good quality of knowledge" (relationship (1); $\Delta = 0{,}5;\ \mu = 0{,}5611$) by the value $\mu_z = 0{,}9908$.
"The average student has a very good knowledge of the solution" (relationship (1); $\Delta = 0{,}5;$ $\mu = 0{,}6429$) by the value $\mu_z = 0{,}9505$.

Furthermore, we can estimate the level of credibility of the statements:
"The average student is successful in terms of assessing the quality of knowledge" (relationship (2); $\Delta = 0;\ \mu = 0{,}5611$) by the value $\mu_u = 0{,}1926$.
"The average student is successful in terms of assessing the knowledge of the solution" (relationship (2); $\Delta = 0;\ \mu = 0{,}6429$) by the value $\mu_u = 0{,}1275$.

We now interpret the average results of individual problems as the fuzzy sets $\overline{A}$ for the quality and $\overline{B}$ for the solution as follows:

$$\overline{A} = \left\{{}^{1}/_{0,6971};\ {}^{2}/_{0,7143};\ {}^{3}/_{0,1986};\ {}^{4}/_{0,6343}\right\}$$
$$\overline{B} = \left\{{}^{1}/_{0,8571};\ {}^{2}/_{0,8571};\ {}^{3}/_{0,1429};\ {}^{4}/_{0,7143}\right\}$$

and determine linear and quadratic indices of fuzziness for them:

$$\vartheta(\overline{A}) = 0{,}5764;\ \eta(\overline{A}) = 0{,}5858;\ \vartheta(\overline{B}) = 0{,}3571;\ \eta(\overline{B}) = 0{,}378.$$

The calculated indices indicate relatively large fuzziness of the fuzzy set $\overline{A}$ and lower fuzziness of the fuzzy set $\overline{B}$, which can be interpreted by higher differences among the solvers in the majority of the problems in terms of the quality of solutions and smaller differences among the solvers in the majority of the problems in terms of the accuracy of the solution. This corresponds to the approach of the students to solving the problems. They are motivated to find solutions but not to understand the subject matter. Variability of thoughts and variability of responses is very high. If we evaluate the test by simply counting the points, this variability unfortunately does not show up, we are able to detect it only through the evaluation vector for each case and through calculating the relevant variability index. It implies that the method of teaching in question does not suit all the students equally.

Table 4 shows the transformed average results obtained after 23 iterations for the quality of the solution and 26 iterations for the solution.

Table 4. Values obtained by Iteration

Problem	Values obtained by Iteration				Overall Average
	1.	2.	3.	4.	
Quality	0,2069	0,2069	0,2068	0,2069	0,2069
Solution	0,1429	0,1429	0,1429	0,1429	0,1429

The data in Table 4 for the iterative average help us estimate the degree of credibility of following statements:
"The average student has a good quality of knowledge" (relationship (1); $\Delta = 0;\ \mu = 0{,}2069$) by the value $\mu_z = 0{,}102$.
"The average student has a good knowledge of the solution" (relationship (1); $\Delta = 0;\ \mu = 0{,}142$) by the value $\mu_z = 0{,}0495$.

Similarly, using the values for the overall average from Table 4, we shall estimate assuming the semantics of the term "not too" given by the parameter $\Delta = 0{,}5$ the level of credibility of the statement:
"The average student has a not too good quality of knowledge" (relationship (1); $\Delta = 0{,}5; \mu = 0{,}2069$) by the value $\mu_z = 0{,}8026$.

"The average student has a not too good knowledge of the solution" (relationship (1); $\Delta = 0{,}5$; $\mu = 0{,}1429$) by the value $\mu_z = 0{,}717$.

Furthermore, we can estimate the level of credibility of the statements:
"The average student is successful in terms of assessing the quality of knowledge" (relationship (2); $\Delta = 0$; $\mu = 0{,}2069$) by the value $\mu_u = 0{,}629$.
"The average student is successful in terms of assessing the knowledge of the solution" (relationship (2); $\Delta = 0$; $\mu = 0{,}1429$) by the value $\mu_u = 0{,}7346$.

We now interpret the average results of individual problems as the fuzzy sets $\overline{A}$ for the quality and $\overline{B}$ for the solution as follows:

$$\overline{A} = \left\{ {}^{1}/_{0,2069};\ {}^{2}/_{0,2069};\ {}^{3}/_{0,2068};\ {}^{4}/_{0,2069} \right\}$$

$$\overline{B} = \left\{ {}^{1}/_{0,1429};\ {}^{2}/_{0,1429};\ {}^{3}/_{0,1429};\ {}^{4}/_{0,1429} \right\}$$

and determine linear and quadratic indices of fuzziness for them:

$$\vartheta(\overline{A}) = 0{,}4138;\ \eta(\overline{A}) = 0{,}4138;\ \vartheta(\overline{B}) = 0{,}2858;\ \eta(\overline{B}) = 0{,}2858.$$

The calculated indices indicate relatively large fuzziness of the fuzzy set $\overline{A}$ and lower fuzziness of the fuzzy set $\overline{B}$, which can be interpreted by higher differences among the solvers in the majority of the problems in terms of the quality of solutions and smaller differences among the solvers in the majority of the problems in terms of the accuracy of the solution.

Conclusion

The great importance of the article presented can be seen in the presented method of the experiment evaluation methodology, which is particularly suitable when the number of respondents is below the required values for the classical statistical processing. It is not suitable to use this methodology only to evaluate the teaching experiment, but in all cases where we measure uncertainty in conditions of vagueness that are due to the absence of a prototype unit of the measured property. There had been demonstrated that especially teaching methods can be easily assessed using the fuzzy characteristics and student results can be accepted with no major alterations.

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