



Fostering cognitive mathematical modeling competencies in 4th-grade students: Cooperative learning strategies

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ABSTRACT

The study aims to explore how 4th-grade students' cognitive mathematical modeling competencies develop during the implementation of model-eliciting activities within a cooperative learning framework. The study was designed as a qualitative teaching experiment involving eight 4th-grade students, with the longitudinal analysis focusing on three purposively selected focal students representing different initial modeling competency levels. Data were collected through six teaching episodes, four clinical interviews, students' written solution papers, solution letters, and audio-video recordings, and analyzed using descriptive analysis of the rubric for the assessment of modelling skills. The findings indicate progressive development in the focal students' cognitive mathematical modeling competencies. In particular, the cooperative learning strategies (think-talk-write, think-pair-share, and timed pair-share) supported students' progression from result-oriented approaches toward more systematic processes of understanding the task, simplifying, mathematizing, interpreting, and validating. These findings have implications for integrating cooperative learning with model-eliciting activities to support mathematical modeling instruction in elementary mathematics education.

Keywords: mathematical modeling, cognitive modeling competence, cooperative learning strategies, elementary school

INTRODUCTION

Mathematical modeling is a cyclical process of mathematizing a real-world situation, obtaining its mathematical result, and interpreting the mathematical result in the context of real life (Lesh & Doerr, 2003). Researchers, professional organizations, and mathematics education standards emphasize the necessity and importance of mathematical modeling, especially from the early school years (Asempapa, 2015; Blum & Borromeo Ferri, 2009). In model-eliciting activities, students engage in a cyclical process of defining real-life problems, developing and solving mathematical models, and interpreting results within the original context. Although mathematics curricula include objectives and outcomes for students' mathematical modeling, most mathematics or classroom teachers rarely try to relate mathematics lessons to students' daily lives (Steen et al., 2007). Therefore, students fail to make connections between mathematics and daily life (Tran & Dougherty,

This study is derived from the first author's PhD thesis, conducted under the supervision of the second and third authors.

2014) and miss the opportunity to apply the knowledge they learn to the problem situations they encounter (Haas et al., 2020).

In the mathematical modeling process, mathematical models are constructed and solutions to real life situations are sought (Lesh & Doerr, 2003). Model-eliciting activities used in this process are non-routine, non-routine, real-life-related problem types that involve different possible solutions, in which students work collaboratively in groups and produce solutions based on many possibilities in order to create a model that can be generalizable and prototype (Lesh & Doerr, 2003). Model-eliciting activities promote the development of flexible problem-solving, metacognitive regulation, critical inquiry, mathematical communication, and the mathematization of real-world phenomena through the transformation of qualitative data into quantitative structures (Canbazoglu-Albayrak & Tarim, 2025; English, 2007).

An examination of studies involving elementary school students indicates that existing research has predominantly focused on 4th-grade students and has mainly examined their mathematical modeling processes and the difficulties they encounter (English, 2006, 2010; Haas et al., 2020; Suh et al., 2017, 2021; Wickstrom & Aytes, 2018). These studies consistently report that students experience challenges in understanding, simplifying, interpreting, and validating modeling tasks. However, considerably less attention has been devoted to identifying instructional approaches that effectively support students in overcoming these difficulties and developing their mathematical modeling competencies.

Extant research has therefore provided valuable insights into what difficulties elementary students experience during mathematical modeling, particularly in moving from real-world situations to mathematical models (English, 2010; Suh et al., 2021). While these descriptive studies establish a foundation for understanding the “what” of student difficulties, there remains a notable dearth of research on the “how”, specifically, the instructional mechanisms required to mediate these processes. Unlike previous studies (e.g., Wickstrom & Aytes, 2018) that view modeling as a standalone task, this study positions the modeling cycle within a structured pedagogical intervention. By integrating cooperative learning strategies, think-talk-write, think-pair-share and timed pair-share, the present study differentiates itself by providing a socio-cognitive scaffold for students. Because the mathematical modeling process is specifically designed for small group work in which students are required to develop clearly shareable products that include explanations, assumptions, justifications and mathematical representations (English & Watters, 2004). While existing research characterizes modeling proficiency as a byproduct of experience (Leiss et al., 2010), this research articulates that modeling competence is a constructible skill nurtured through structured peer-mediated discourse, thereby addressing the gap in instructional scaffolding for young learners.

Unlike previous studies that primarily documented students’ modeling processes or the difficulties they encountered, the present study investigates how cognitive mathematical modeling competencies can be developed through the deliberate integration of model-eliciting activities and cooperative learning strategies within a longitudinal teaching experiment. The integration of these approaches is pedagogically meaningful because model-eliciting activities naturally require students to discuss ideas, justify assumptions, negotiate alternative solutions, and communicate mathematical representations while working collaboratively (English & Watters, 2004). In addition to addressing this empirical gap, the present study contributes to the literature by providing evidence on how cooperative learning can support the development of elementary students’ cognitive mathematical modeling competencies during model-eliciting activities. Rather than focusing solely on students’ final modeling performance, the study examines how specific cognitive competencies evolve throughout successive model-eliciting activities. In doing so, it extends current understanding of how instructional design can facilitate the development of mathematical modeling competencies in elementary education.

Previous research has also shown that students often bypass systematic modeling stages in favor of immediate, result-oriented solutions, which limits the quality of the mathematical models they develop (Blum & Borromeo-Ferri, 2009; Greefrath, 2015). Furthermore, students frequently fail to critically interpret and validate their mathematical results within real-life contexts (Blum & Borromeo-Ferri, 2009; Sekerak, 2010), underscoring the need for instructional approaches that explicitly support these aspects of the modeling process. Responding to these gaps, the present study explores the development of 4th-grade students’ cognitive mathematical modeling competencies through model-eliciting activities implemented within a

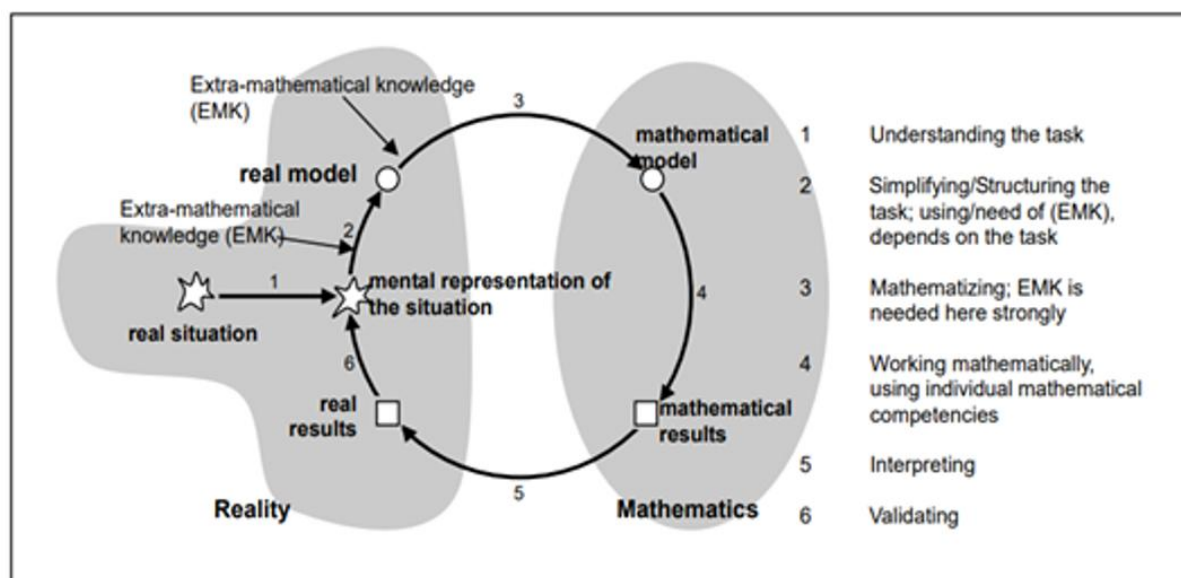


Figure 1. Modeling cycle under cognitive perspective (Borromeo Ferri, 2006)

cooperative learning framework. To provide a fine-grained analysis of this developmental process, the study follows the longitudinal progress of three representative students across a series of teaching episodes. Accordingly, the following research question guided the study: How do 4th-grade students demonstrate and develop their cognitive mathematical modeling competencies during the implementation of model-eliciting activities within a cooperative learning framework? To provide a high-resolution analysis of this developmental process, the study focuses on the longitudinal progress of three representative students throughout a series of teaching episodes.

THEORETICAL FRAMEWORK

Modeling Cycle Under Cognitive Perspective

Mathematical modeling competence is defined as the ability to identify variables and assumptions in real-world situations, translate them into mathematical terms, and critically validate the outcomes (Niss et al., 2007). Modeling cycles are frequently used to operationalizing these competencies (Blomhøj, 2011). This study adopts Borromeo Ferri's (2006) cognitive modeling cycle, which consists of six interrelated phases: understanding the task, simplifying, mathematizing, working mathematically, interpreting, and validating (see [Figure 1](#)).

In this cycle, students first construct a mental representation by identifying given and desired information. The transition to a “real model” involves structuring the situation through simplifying assumptions. During the mathematization phase, these assumptions are transformed into mathematical models (e.g., equations or graphs), which are subsequently solved during the mathematical working phase to obtain formal results. Finally, students interpret these results within the real-world context and validate them against their initial mental models and real-life experiences. If inconsistencies are identified during validation, students cyclically revisit the relevant stages to refine their models.

The Importance of Mathematical Modeling in Elementary Classrooms

Peter-Koop (2004) emphasizes that students' understanding of the task is important for solving the problem and that they generally have difficulty at this stage of the modeling process. However, according to Blum and Borromeo-Ferri (2009) and Haas et al. (2020), since students do not encounter model-eliciting activities both in and out of school, they proceed directly to the solution process without thinking about the problem situations and understanding the problem. In many studies conducted with students from different age groups, it has been emphasized that simplifying the problem situation, which is one of the first stages of the modeling cycle, is quite challenging (Blum, 2015; Chan et al., 2012; Haines & Crouch, 2007; Maaß, 2006;

Table 1. Cooperative learning strategies and mathematical modeling competencies

Cooperative learning strategy	Phase	Cognitive mathematical modeling competencies	Theoretical rationale
Think-talk-write	Think	Understanding the task	Students individually analyze the problem, identify relevant information and activate prior knowledge before discussion.
	Talk	Simplifying, mathematizing, interpreting	Collaborative discussion supports negotiation of assumptions, interpretation of the problem and construction of mathematical representations.
	Write	Mathematizing, working mathematically, validating	Writing encourages students to justify, communicate and evaluate their mathematical models.
Think-pair-share	Think	Understanding the task	Individual thinking allows students to formulate initial ideas before peer interaction.
	Pair	Simplifying, mathematizing, interpreting	Peer discussion promotes reasoning, comparison of ideas and refinement of mathematical models.
	Share	Mathematizing, working mathematically, validating	Whole-class sharing enables explanation, justification and evaluation of alternative solutions.
Timed pair-share	Think	Understanding the task	Individual reflection supports comprehension of the modeling task.
	Pair	Simplifying, mathematizing, interpreting	Equal discussion time ensures active participation and collaborative reasoning.
	Share	Mathematizing, working mathematically, validating	Presenting and discussing solutions encourages validation and reflection.

Sekerak, 2010) and that students have difficulty in creating assumptions appropriate to the problem situation (Blum, 2015). This situation was also emphasized in Maaß's (2006) study and it was stated that students did not discuss during the modeling process, a significant part of the solution assumptions they put forward were incomplete and this situation affected the competencies that would emerge during the discussion. In the mathematizing process, it has been emphasized that although students are mathematically well-equipped, they put forward models in which they prefer simpler mathematical operations (Biccard & Wessels, 2011; Blum, 2011). It is emphasized that in the process of working mathematically, students want to proceed directly to solving the problem and cannot develop a systematic solution path or put forward a model (Blum & Borromeo Ferri, 2009; Haas et al., 2020). It is stated that students' desire to quickly develop result-oriented solutions negatively affects the problem-solving process in modeling studies (Greefrath, 2015). In addition, it is emphasized that students do not interpreting and validating the appropriateness of the mathematical results they obtain for real life situations (Blum & Borromeo-Ferri, 2009; Peter-Koop, 2004; Sekerak, 2010). Considering that the steps of interpreting solutions and verifying the model are accepted as important steps for the modeling process (Blum & Borromeo-Ferri, 2009), it is thought that it is important to improve these aspects of students. For this reason, in this study, the development of cognitive mathematical modeling competencies of 4th-grade elementary school students was examined by applying model-eliciting activities within the scope of "think-talk-write", "think-pair-share", and "timed pair-share" cooperative learning strategies. The relationship between these strategies and cognitive mathematical modeling competencies is explained in the next subheading.

Cooperative Learning Strategies

Think-talk-write, think-pair-share, and timed pair-share strategies are the cooperative learning strategies used in this study. The relationship between the stages of these strategies and cognitive mathematical modeling competencies is explained in [Table 1](#).

The mappings presented in [Table 1](#) were developed based on the theoretical characteristics of each cooperative learning strategy and the cognitive mathematical modeling competency framework adopted in this study. The "think" phase emphasizes students' individual comprehension of the problem and therefore primarily supports understanding the task. The collaborative discussion phases (talk/pair) encourage students to negotiate assumptions, interpret information, simplify real-world situations and construct mathematical representations, thereby supporting simplifying, mathematizing and interpreting competencies. Finally, the Write and Share phases require students to communicate, justify and evaluate their mathematical models, fostering working mathematically and validating competencies. The repeated placement of *mathematizing* in [Table 1](#) does not indicate two separate scoring instances or double counting. Rather, it reflects the

complementary cognitive functions of the cooperative learning phases in supporting the same modeling competency. The talk/pair phase primarily supports students in developing, negotiating, and refining mathematical representations and relationships through peer discussion, whereas the Write/Share phase supports students in formalizing, communicating, and further evaluating these mathematical constructions.

The think-talk-write strategy is based on the understanding that learning is a social behavior (Pratiwi, 2019). It encourages students to think about a topic, talk about it, and then write about it. When students are given written tasks, problems, or assignments in learning activities, they invariably begin directly solving the problem or writing down their answers (Indahyanti, 2017). However, before moving on to the problem-solving process, it is even more crucial for students to think, generate, develop, and test their ideas for learning and making sense of them (Astuti, 2014; Indahyanti, 2017). In this context, the think-talk-write strategy provides students with rich and diverse opportunities to engage in these activities (thinking, reflecting, organizing ideas, and testing them before writing). Thus, the think-talk-write strategy plays an important role in students' development of mathematical problem-solving skills (Guswinda et al., 2019; Huda et al., 2020; Pratiwi, 2019; Susanto et al., 2018; Wahyuni et al., 2020). By using the think-pair-share and timed pair-share strategies, students recall their prior knowledge during the individual idea generation process and have the opportunity to generate new ideas using this prior knowledge (Rosyida et al., 2016; Sharma & Saarsar, 2018). In this process, they can experience opportunities to generate and analyze ideas, participate in small group interactions, synthesize information, develop and apply reading skills. It also contributes to the development of higher-order thinking skills and allows students to reason individually before sharing their ideas with others (Kothiyal et al., 2013). Therefore, because mathematical modeling is a problem-solving process, we can say that these strategies will contribute to the development of students' mathematical modeling competencies. It has been emphasized that these strategies are among the learning strategies aimed at improving students' problem-solving abilities (Afthina & Pramudya, 2017).

These strategies are basically realized through thinking, talking (discussion), writing and presenting. **Table 1** illustrates how these stages map onto the cognitive modeling cycle. First, students are divided into groups of 4-6 students. Then the teacher gives the students a problem situation in a real life context. Students individually read the problem situation in the group, take notes and think about possible answers (solution strategies) individually. Thus, students realize the thinking stage, in other words, the competence of understanding the task. Then, students discuss their ideas with their group mates. In this process, students put their skills of evaluating ideas, forming solution hypotheses, critical thinking and reasoning to work. With the talking (discussion) phase, students actually realize the competencies of simplifying, mathematizing and interpreting the problem by creating a solution hypothesis. In the writing phase, students apply the solution hypothesis they have created and perform the competencies of mathematizing, working mathematically and validating. The last learning activity is the stage of presenting their solutions in the form of a written report. While one student from each group presents their solutions, the other groups critically evaluate the solutions and generate ideas. In addition, the final stage of the mathematical modeling process, the presentation competency, is also realized. Finally, the implementation stages of these strategies also allow students to realize a cyclical process. In this context, since model-eliciting activities by their nature require students to work collaboratively in groups, understand the problem situation, simplify it, formulate a model, analyze it, interpret and verify the result, and report the result in writing, it is thought that students' having thinking, talking (discussion), writing and presenting behaviors in this process will improve their modeling competencies. In this context, the development of cognitive mathematical modeling competencies of 4th-grade elementary school students was examined in this study by applying model-eliciting activities with cooperative learning strategies.

Modeling and Cooperative Learning: A Cognitive Relationship

The cooperative learning strategies adopted in this study not only provide the instructional structure for implementing model-eliciting activities but also offer a conceptual mechanism for supporting the development of students' cognitive mathematical modeling competencies. Rather than viewing cooperative learning and mathematical modeling as two separate instructional approaches, this study conceptualizes cooperative learning as a pedagogical scaffold that supports students throughout the cognitive processes involved in Borromeo Ferri's (2006) modeling cycle. Specifically, the individual thinking, collaborative

discussion, and sharing phases of think-talk-write, think-pair-share, and timed pair-share are expected to facilitate different cognitive mathematical modeling competencies.

The individual *thinking* phase primarily supports the competencies of understanding the task. Previous research has shown that elementary students often move directly toward computation without sufficiently analyzing the problem situation (Blum & Borromeo-Ferri, 2009; Greefrath, 2015). In the cooperative learning strategies employed in this study, students first examine the problem individually, identify relevant information, and organize their initial ideas before interacting with their peers (Astuti, 2014; Sharma & Saarsar, 2018). This deliberate reflection encourages students to develop a clearer understanding of the problem context and to distinguish essential information from irrelevant details, thereby supporting the initial stages of the modeling cycle (Peter-Koop, 2004).

The collaborative *talk and pair* phases are expected to foster the competencies of simplifying, mathematizing and interpreting. During discussion, students explain their ideas, compare alternative solution strategies, negotiate assumptions, and justify their reasoning. Such interactions encourage students to translate real-world situations into mathematical representations while simultaneously interpreting the meaning of their emerging mathematical models (Biccard & Wessels, 2011; Maaß, 2006). Through peer interaction, students are also exposed to multiple perspectives, allowing them to refine their interpretations and revise their mathematical thinking when necessary.

Finally, the *writing and sharing* phases support the competencies of mathematizing, working mathematically and validating. Writing requires students to organize and communicate their mathematical reasoning, whereas sharing their solutions with peers encourages them to justify their models, respond to questions, and evaluate the appropriateness of their solutions (Astuti, 2014; Rosyida et al., 2016). These processes promote reflection on both the mathematical procedures used and the suitability of the resulting models within the original real-world context, which are essential characteristics of validation in mathematical modeling (Blum & Borromeo-Ferri, 2009).

Accordingly, cooperative learning is viewed in the present study as a pedagogical scaffold that supports students throughout the modeling cycle. By structuring opportunities for individual reflection, collaborative reasoning, and collective evaluation, the integration of model-eliciting activities with cooperative learning strategies is expected to facilitate the progressive development of elementary students' cognitive mathematical modeling competencies.

METHOD

Research Design

In the research, the question of how to ensure the development of students' cognitive mathematical modeling competencies comes to the forefront. Accordingly, in this study, the teaching experiment, which is included in qualitative research and is especially used in mathematics education to investigate how students learn as a whole and to systematically examine what happens, was chosen as the research design (Czarnocha & Maj, 2008). The main purpose of the teaching experiment is to closely experience the process of students' constructing mathematical knowledge and to build models that explain how student mathematics changes in designed learning environments (Cobb & Steffe, 2011). In a teaching experiment, it can be determined which variables are effective in students' learning and how they integrate their previous knowledge with new knowledge and how they construct this new knowledge (Steffe & Ulrich, 2014). For this reason, it is stated that using the teaching experiment method is one of the most appropriate and effective ways for a research focusing on learning processes (Cobb & Steffe, 2011).

The selection of a teaching experiment was guided by the purpose of the present study. The study sought to examine how 4th-grade students' cognitive mathematical modeling competencies developed during the implementation of model-eliciting activities within a cooperative learning environment. Addressing this research question required an approach capable of capturing gradual changes in students' reasoning across successive instructional episodes rather than documenting performance at a single point in time. The teaching experiment design therefore provided an appropriate methodological framework for tracing the development of the six cognitive mathematical modeling competencies and for examining how interactions

within cooperative learning activities supported this developmental process. Although other qualitative approaches, such as case studies or design-based research, can provide valuable insights into educational settings, the primary purpose of this study was not to describe a bounded case or to iteratively refine an instructional intervention. Instead, the focus was on understanding students' cognitive development within an intentionally designed learning environment. For this reason, a teaching experiment was considered the most appropriate research design for addressing the research question.

Procedure

The teaching experiment consisted of teaching episodes and clinical interviews (Steffe & Ulrich, 2014). The intervention phase comprised six teaching episodes in which two heterogeneous cooperative learning groups, each consisting of four students, engaged in model-eliciting activities implemented through the think-talk-write, think-pair-share, and timed pair-share strategies. To monitor the development of students' cognitive mathematical modeling competencies, individual clinical interviews were conducted with the three focal students at four points during the study: one pre-clinical interview, two mid-clinical interviews, and one final clinical interview. This longitudinal implementation enabled the researchers to examine changes in students' cognitive mathematical modeling competencies across the intervention. All teaching episodes and clinical interviews were audio- and video-recorded for subsequent analysis.

Researcher's role and reflexivity

In this teaching experiment, the first author assumed the role of a "researcher-teacher" actively participating in the classroom to observe students' construction of mathematical knowledge within a natural learning environment (Cobb & Steffe, 2011). Consistent with the constructivist perspective underlying teaching experiments, the researcher acted as a facilitator rather than a direct instructor. During the implementation of the model-eliciting activities, interactions with students were limited to open-ended, non-leading questions designed to encourage them to explain, justify, and reflect on their own thinking. The researcher deliberately avoided evaluating students' responses or directing them toward particular solution strategies, thereby allowing students' mathematical reasoning to develop naturally throughout the modeling process. In addition, all activities were implemented according to a predetermined instructional plan to ensure consistency across teaching episodes.

To minimize researcher bias and enhance the reflexivity of the study, several methodological procedures were employed. First, data triangulation was achieved through the use of multiple data sources, including model-eliciting activity sheets, students' written solution papers, solution letters, video and audio recordings, interview transcripts, and field notes. Second, regular peer debriefing sessions were conducted with an expert in mathematics education and a qualitative research specialist throughout both the data collection and analysis processes. These discussions challenged emerging interpretations, questioned potential researcher assumptions, and helped ensure that analytical decisions remained grounded in the empirical data rather than the researcher's expectations.

Furthermore, to reduce subjective influence during data analysis, the qualitative coding process was independently conducted by two researchers in addition to the first author. The coding results were then compared, discussed, and refined until consensus was reached. Inter-coder reliability was calculated using the Miles and Huberman (2016) formula, demonstrating a high level of agreement among the coders. Finally, ongoing and retrospective analyses of the audio-visual recordings were conducted to monitor changes in students' mathematical thinking and to ensure that interpretations accurately reflected the progression of students' cognitive mathematical modeling competencies throughout the teaching experiment (Steffe & Ulrich, 2014).

Study Group

The study involved eight 4th-grade students from a public school, initially organized into two heterogeneous cooperative learning groups. To determine these groups and identify the focus participants, students were assessed using the "gasoline filling" problem and scored via the rubric for the assessment of modelling skills (RAMS) (0-25 points). Proficiency levels were defined as low (0-8), medium (9-17), and high (18-25). Two four-person groups were formed with a heterogeneous structure to allow students with varying

Table 2. Student scores and groupings in teaching episodes

Group 1	Group 2
Dolphin* (18 points)	Lilium (18 points)
Lemur* (11 points)	Orchid flower (8 points)
Cactus (0 points)	Snapdragon* (0 points)
Sunflower (4 points)	Dahlia (0 points)

* The students selected as focus participants

Table 3. Personal information of the study group

Participant	DOB	Pre-school education	Mother's education	Father's education	Mathematics achievement
Snapdragon	02.14.2013	Yes	High school graduate	High school graduate	Good
Lemur	10.17.2013	Yes	University graduate	University graduate	Good
Dolphin	05.13.2013	Yes	High school graduate	University graduate	Good

proficiency levels to work together. Since the participants had no prior experience with modeling, this interactive grouping aimed to foster diverse solution approaches and enhance their modeling competencies through peer-mediated learning. Students were assigned the code names Cactus, Snapdragon, Dahlia, Sunflower, Orchid Flower, Lemur, Lilium, and Dolphin. Detailed scores and group distributions are presented in [Table 2](#).

From this pool, three focus participants were selected for longitudinal clinical interviews to capture a broad developmental spectrum: Snapdragon (low, 0 points), Lemur (medium, 11 points), and Dolphin (high, 18 points). These focus students participated in individual clinical interviews alongside the group-based teaching episodes, allowing for a detailed tracking of their cognitive mathematical modeling competencies. While the small sample size is a limitation, this case-oriented approach was essential for the high-resolution analysis required by the teaching experiment. Profiles of these three students are detailed in [Table 3](#).

From the eight participants, three focal students, Snapdragon (low), Lemur (medium), and Dolphin (high), were purposively selected for individual clinical interviews based on their initial mathematical modeling competency levels. The selection was based on their initial modeling competency levels as determined by the RAMS assessment. Choosing one student from each proficiency level enabled the study to capture diverse developmental trajectories and examine how students with different initial competencies responded to the same instructional intervention. The purpose of this selection was not to achieve statistical representativeness but to obtain information-rich cases capable of providing an in-depth understanding of the development of cognitive mathematical modeling competencies (Creswell & Poth, 2016; Patton, 2015).

This purposive sampling strategy is consistent with the objectives of teaching experiments, which seek to explain how students' mathematical thinking develops over time within carefully designed instructional environments rather than to generalize findings to larger populations (Cobb & Steffe, 2011). Because the three focal students participated in all teaching episodes and clinical interviews, their complete longitudinal data allowed the researchers to examine changes in each cognitive modeling competency across the intervention and to construct detailed developmental accounts of students' reasoning processes.

Data Collection

Data were collected through task-based interviews, a method designed to capture students' evolving mathematical knowledge and problem-solving behaviors (Goldin, 2000). In these interviews, students interact with carefully designed task environments, enabling researchers to obtain in-depth insights into their cognitive processes (Maher et al., 2011). In the present study, eleven model-eliciting activities were developed by the first author for 4th-grade elementary school students.

The development of the model-eliciting activities followed a systematic multi-stage process to ensure both theoretical and content validity. First, the activities were designed in accordance with the model-eliciting activity design principles proposed by Lesh et al. (2000), including the reality principle, model construction principle, self-assessment principle, construct documentation principle, construct shareability and reusability principle, and effective prototype principle. In addition, the activities were aligned with the learning outcomes of the 4th-grade mathematics curriculum and situated within authentic real-life contexts that were familiar and meaningful to elementary school students.

Following the initial development, the activities were reviewed by an expert panel consisting of two mathematics education specialists and one curriculum specialist. The experts evaluated each activity in terms of its alignment with the curriculum objectives, consistency with the principles of model-eliciting activities, clarity of the problem situations, cognitive demand, and its potential to elicit students' cognitive mathematical modeling competencies. Based on their feedback, revisions were made to improve the clarity of the task instructions, refine the contextual scenarios, adjust the cognitive demand of the activities, and ensure that the sequence of tasks progressed from relatively simple to more cognitively demanding modeling situations.

Subsequently, a pilot study was conducted with a group of 4th-grade students who did not participate in the main study. The pilot implementation aimed to examine the feasibility of the activities, identify ambiguities in the task instructions, evaluate the appropriateness of the allocated time, and determine whether the activities successfully engaged students in the intended mathematical modeling processes. Following the pilot implementation, minor revisions were made to simplify the wording of several task instructions and clarify contextual information that students found difficult to interpret.

The eleven model-eliciting activities served different purposes throughout the research process. One activity was used to determine students' initial mathematical modeling competency levels and to establish the cooperative learning groups. Four activities were administered during the pre-, mid-, and post-clinical interviews, whereas the remaining six activities were implemented during the teaching episodes. The activities were intentionally sequenced from lower to higher cognitive demand in order to progressively support the development of students' cognitive mathematical modeling competencies. Throughout the implementation, students' modeling processes were documented through audio and video recordings, written solution papers, solution letters, and clinical interview transcripts. The use of these multiple data sources enabled a comprehensive examination of students' cognitive mathematical modeling competencies. Information about the model-eliciting activities is presented in [Table 4](#).

The model-eliciting activities were intentionally sequenced from lower to higher levels of cognitive demand. Early activities emphasized understanding problem situations, identifying relevant information, and organizing data, whereas subsequent activities progressively required students to integrate multiple mathematical concepts, construct mathematical models, justify their reasoning, and validate their solutions. This progression was designed to support the gradual development of students' cognitive mathematical modeling competencies throughout the teaching experiment.

The teaching experiment followed a structured implementation consisting of six teaching episodes and four clinical interview phases, as detailed in [Table 4](#). The teaching episodes, involving cooperative group work, were conducted in two-hour class sessions. During these episodes, students were organized into two heterogeneous groups of four to engage in collaborative model-eliciting activities. To monitor individual progress, face-to-face clinical interviews were conducted with the focus participants at four distinct intervals: pre-clinical, two mid-clinical, and final clinical interviews.

As outlined in [Table 4](#), these interviews were strategically timed relative to the teaching episodes to establish baselines and track developmental shifts. During these audio-visual recorded sessions, task-based questions were employed to elicit deep cognitive mathematical modeling competencies (e.g., What did you understand from the problem?, How do you know your solution is correct?, etc.).

Data Analysis

The data were analyzed using descriptive analysis, a qualitative approach in which data are organized and interpreted according to predetermined themes (Miles & Huberman, 2016). In the present study, the analytical framework was established a priori through the RAMS (Tekin-Dede & Bukova-Güzel, 2018). Accordingly, a deductive coding approach was adopted, whereby students' responses were analyzed according to the six cognitive mathematical modeling competencies defined by Borromeo Ferri (2006): understanding the task, simplifying, mathematizing, working mathematically, interpreting, and validating. Rather than generating categories inductively from the data, all qualitative evidence was examined within this predetermined analytical framework.

To systematically evaluate students' modeling competencies, the RAMS was employed as the analytical scoring rubric. The RAMS assesses students' performance across the six cognitive mathematical modeling

Table 4. Data collection

Model eliciting activities	Sources of model eliciting activities	Description of the activity	Activity type	Duration
Gasoline filling	Inspired by Blum and Borromeo Ferri (2009)	Students determine the most appropriate fueling strategy by organizing and interpreting numerical information presented in a real-life context.	Determining the study group and students in focus	32'
Why BIG-S burger?	Adapted from Doerr and English (2003)	Students compare alternative purchasing options and develop a mathematical model to justify the most reasonable choice.	Pre-clinical interview	S: 24'59" L: 21'08" D: 19'49"
Little black fish	Inspired by Maaß et al. (2018)	Students analyze a real-life situation by identifying relevant information and constructing a mathematical solution to support decision-making.	Teaching episode 1	55'08"
Decorating my room!	Inspired by Maaß et al. (2018)	Students design a room layout by integrating measurement concepts and mathematical calculations while satisfying realistic spatial constraints.	Teaching episode 2	70'09"
Time at work	Inspired by Maaß and Mischo (2011)	Students develop a mathematical model to organize work schedules by coordinating numerical relationships involving time.	First mid-clinical interview	S: 25'07" L: 29'56" D: 24'45"
Robot cleaner	Original model eliciting activities	Students evaluate ten robot cleaner alternatives by analyzing quantitative information and develop a mathematical model to justify which option represents the most appropriate choice.	Teaching episode 3	76'43"
Which computer engineer?	Original model eliciting activities	Students evaluate alternative candidates using quantitative information and justify their decision through mathematical reasoning.	Teaching episode 4	68'52"
Tennis tournament	Inspired by Doerr and English (2003)	Students organize tournament matches by developing an efficient mathematical model that satisfies the given conditions.	Second mid-clinical interview	S: 26'46" L: 23'24" D: 20'24"
Big foot	Adapted from Lesh and Doerr (2003)	Students estimate and justify unknown measurements using proportional reasoning and mathematical representations in a real-life context.	Teaching episode 5	43'09"
Painting our classroom!	Inspired by Maaß and Mischo (2011)	Students estimate the amount of paint required to paint their classroom by measuring the classroom walls and constructing a mathematical model based on real-life measurement data.	Teaching episode 6	44'16"
Adenauer statue	Adapted from Herget et al. (2000)	Students estimate the dimensions of a statue by constructing, testing, and validating mathematical models based on available information.	Final clinical interview	S: 24'01" L: 22'34" D: 24'17"

Note. S: Snapdragon; L: Lemur; D: Dolphin

competencies using performance descriptors representing increasing levels of proficiency. Total scores range from 0 to 25 and are classified as follows: not having modeling competence (0-6), having modeling competence to some extent (7-12), having acceptable modeling competence (13-21), and having high level modeling proficiency (22-25).

The detailed performance descriptors for each competency are presented in [Table 5](#). Although RAMS assigns numerical scores to its levels, the rubric does not represent a strictly linear progression of competence. In particular, some levels in mathematizing and validating reflect different qualitative aspects of performance rather than uniformly increasing proficiency. Therefore, fluctuations in RAMS scores were not interpreted as direct evidence of competence regression; instead, score changes were considered together with students' verbal, written, and observed modeling behaviors.

The analysis was conducted through a systematic four-stage coding procedure (Miles & Huberman, 2016). *First*, all audio and video recordings obtained during the clinical interviews and teaching episodes were transcribed verbatim. Students' written solution papers, solution letters, field notes, and observation records were then reviewed alongside the transcripts to establish a comprehensive representation of each student's mathematical modeling process.

Second, meaningful units of data were identified from students' verbal explanations, written responses, and observed modeling behaviors. Each unit of evidence was deductively assigned to one of the six cognitive mathematical modeling competencies defined in the RAMS. Within each competency, the identified evidence

Table 5. RAMS (Tekin-Dede & Bukova-Güzel, 2018, p. 54-55)

Level	Definition
Understanding the task	
Level 1 (0 point)	Not understanding the task, not determining the givens and goals, and not forming or mistakenly forming a relationship between them.
Level 2 (1 point)	Understanding the task partly, determining the givens and goals to some extent but not forming or mistakenly forming a relationship between them.
Level 3 (2 point)	Understanding the task completely, determining the givens and goals, not forming or mistakenly forming a relationship between them.
Level 4 (3 point)	Understanding the task completely, making unimportant mistakes in determining the givens and goals, not forming a relationship between them.
Level 5 (4 point)	Understanding the task completely, determining the givens and goals, and forming a relationship between them.
Simplifying	
Level 1 (0 point)	Not simplifying the problem, not determining the necessary/unnecessary variables, and making wrong assumptions.
Level 2 (1 point)	Simplifying the problem partly, determining the necessary/unnecessary variables to some extent, and making wrong assumptions.
Level 3 (2 point)	Simplifying the problem, determining the necessary/unnecessary variables, and making partially acceptable assumptions.
Level 4 (3 point)	Simplifying the problem, determining the necessary/unnecessary variables, and making realistic assumptions.
Mathematizing	
Level 1 (0 point)	Not constructing or mistakenly constructing mathematical model(s).
Level 2 (1 point)	Constructing incomplete/wrong mathematical model(s) based on partially acceptable assumptions.
Level 3 (2 point)	Constructing correct mathematical model(s) based on partially acceptable assumptions.
Level 4 (3 point)	Constructing incomplete/wrong mathematical model(s) based on realistic assumptions and relating them to one another.
Level 5 (4 point)	Correctly constructing the needed mathematical model(s) according to realistic assumptions, explaining model(s) and relating them to one another.
Working mathematically	
Level 1 (0 point)	Not presenting a mathematical solution, solving the constructed models wrongly, or trying to solve the wrong mathematical model.
Level 2 (1 point)	Including deficiencies/mistakes in the solution of the mathematical models constructed incompletely/wrongly.
Level 3 (2 point)	Solving correctly the mathematical models constructed incompletely/wrongly.
Level 4 (3 point)	Including deficiencies/mistakes in the solution of the correctly constructed mathematical models.
Level 5 (4 point)	Achieving the correct mathematical solution by solving the correctly constructed mathematical models.
Interpreting	
Level 1 (0 point)	Misinterpreting or not interpreting the obtained mathematical solution in a real-life context.
Level 2 (1 point)	Incompletely interpreting the erroneous/incomplete mathematical solution in a real-life context.
Level 3 (2 point)	Correctly interpreting the erroneous/incomplete mathematical solution in a real-life context.
Level 4 (3 point)	Incompletely interpreting the obtained correct mathematical solution in a real-life context.
Level 5 (4 point)	Correctly interpreting the obtained correct mathematical solution in a real-life context.
Validating	
Level 1 (0 point)	Not validating or making a wrong validating.
Level 2 (1 point)	Validating partially, not correcting the determined mistakes.
Level 3 (2 point)	Validating partially, correcting the determined mistakes to some extent.
Level 4 (3 point)	Validating partially, correcting the determined mistakes.
Level 5 (4 point)	Validating completely, not correcting the determined mistakes.
Level 6 (5 point)	Validating completely, correcting the determined mistakes to some extent.
Level 7 (6 point)	Validating completely, correcting the determined mistakes.

was compared with the corresponding performance descriptors in the rubric to determine the most appropriate performance level. Scores were assigned only when the students' responses demonstrated the characteristics specified in the relevant RAMS level. In this way, every score was explicitly supported by observable evidence obtained from multiple data sources.

Third, investigator triangulation was employed to enhance the credibility of the analysis. The first author and two independent researchers with expertise in mathematics education independently coded the qualitative data using the RAMS criteria. The coding results were subsequently compared, and any discrepancies were discussed until consensus was reached. Inter-coder reliability was calculated using the

Table 6. Coding framework for cognitive mathematical modeling competencies

Competency	Student evidence	Initial code	RAMS level	Score
Understanding the task	Snapdragon: My teacher says the most important thing here. In other words, he wants us to rank them in order from the most popular to the least popular. We will explain this in a letter. Now I will look at the results of the survey.	Identifies the givens and goals of the problem	Level 3	2
Simplifying	Dolphin: I'm making a table, teacher. I will write about the height, length and width of our feet.	Constructing a table	Level 4	3
Mathematizing	Lemur: Teacher, we gave number 1 to the higher one and ranked it that way. Dolphin: We take it all into consideration. Researcher-teacher: What are you going to do then? Lemur: I think we need to addition all their numbers.	Formulating incorrect mathematical model(s)	Level 2	1
Working mathematically	Researcher-teacher: Now can you tell me verbally what you did? Lemur: First of all, I considered the number of sunny days, the number of days below 15 °C, the number of days above 30 °C, and the average annual rainfall. I gave points to all the cities. Then I added up their points and divided them into groups of three.	Solving mathematical models correctly	Level 5	4
Interpreting	Snapdragon: Now he wants us to rank them. From most important to least important. Based on this price list, children's menus come first, price points second, fast food third, hamburgers fourth, fries fifth. Now we have to explain them in order in a letter. Researcher/teacher: Ok (He shows the order by writing it down). Snapdragon: It's over, teacher.	Failing to interpret the obtained mathematical solution	Level 1	0
Validating	Lemur: Just in case, let me check (Student checks his mathematical operations). I think it's over, teacher.	Validates the mathematical solution by checking the mathematical operations	Level 5	4

Miles and Huberman (2016) agreement formula, yielding an agreement coefficient of 86%, indicating a high level of coding consistency.

Finally, representative quotations and written responses were selected to illustrate how students' statements and behaviors corresponded to specific RAMS performance levels. Table 6 presents examples of the analytical process by demonstrating how students' verbal explanations and written work were linked to the six cognitive mathematical modeling competencies, the corresponding performance levels, and the assigned scores. This transparent evidentiary mapping strengthened the trustworthiness of the analysis by ensuring that all interpretations were directly grounded in the empirical data.

Validity and Reliability

The trustworthiness of the study was established according to the criteria proposed by Lincoln and Guba (1985): credibility, transferability, dependability, and confirmability.

Credibility was enhanced through methodological triangulation, investigator triangulation, and peer debriefing. Multiple sources of evidence, including video and audio recordings, clinical interview transcripts, students' written solution papers, solution letters, and field notes, were analyzed to corroborate emerging interpretations. In addition, the qualitative data were independently coded by the first author and two researchers with expertise in mathematics education using the RAMS framework. Intercoder reliability was calculated using the Miles and Huberman (2016) agreement formula, yielding an agreement coefficient of 86%, which indicates a high level of coding consistency. Any coding disagreements were discussed until consensus was reached. Throughout the research process, the study design, coding decisions, and emerging interpretations were also regularly discussed with experts in mathematics education and qualitative research through peer debriefing sessions to challenge assumptions and strengthen the credibility of the findings.

Transferability was supported by providing rich descriptions of the research context, participant characteristics, instructional process, cooperative learning environment, and model-eliciting activities, enabling readers to determine the applicability of the findings to similar educational settings.

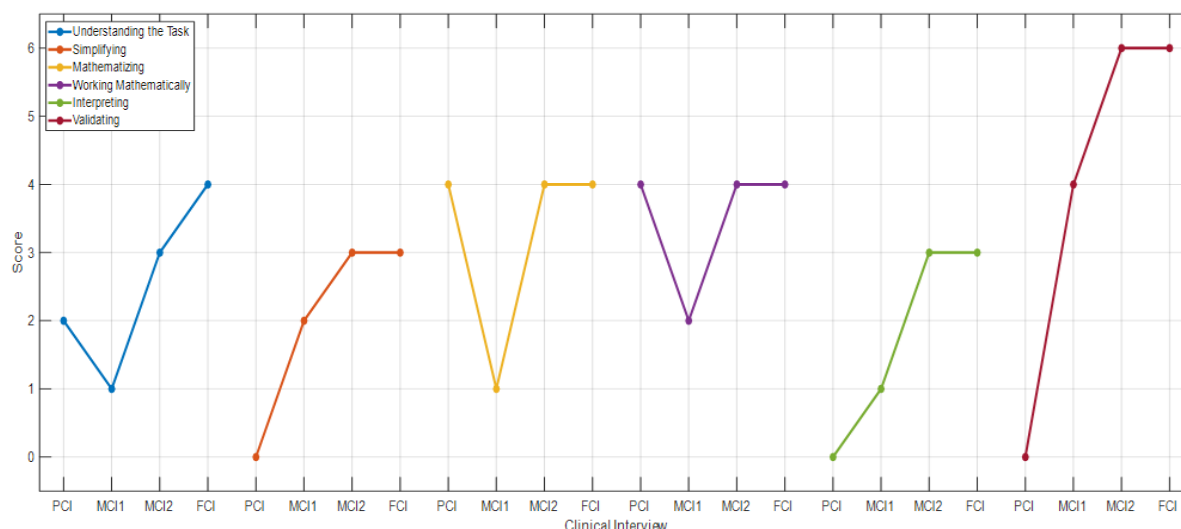


Figure 2. Snapdragon's development of cognitive mathematical modeling competence (PCI: Pre-clinical interview; MCI1: First mid-clinical interview; MCI2: Second mid-clinical interview; FCI: Final clinical interview) (the authors' own elaboration)

Dependability was enhanced by maintaining a transparent audit trail documenting the research procedures, instructional implementation, data collection process, coding decisions, and analytical procedures. This documentation ensured that the research process was systematic, transparent, and open to external review.

Confirmability was addressed by acknowledging the first author's dual role as researcher and teacher and implementing strategies to minimize subjective influence. All interpretations were grounded in multiple sources of empirical evidence, and analytical decisions were continuously reviewed by the research team to ensure that the findings reflected the participants' mathematical modeling processes rather than the researcher's personal expectations.

Ethical Considerations

The study was approved by the Ethics Committee of Çukurova University (Date. 08/04/2022, Approval No. 367357). Prior to data collection, official permission to conduct the research was obtained from the school administration. Written informed consent was obtained from the parents or legal guardians of all participating students through a parental consent form, and written assent was obtained from the students through a voluntary participation form. Participants and their parents were informed about the purpose of the study, the voluntary nature of participation, their right to withdraw from the study at any time without penalty, and the confidentiality of the collected data. To protect participants' identities, pseudonyms were used throughout the study.

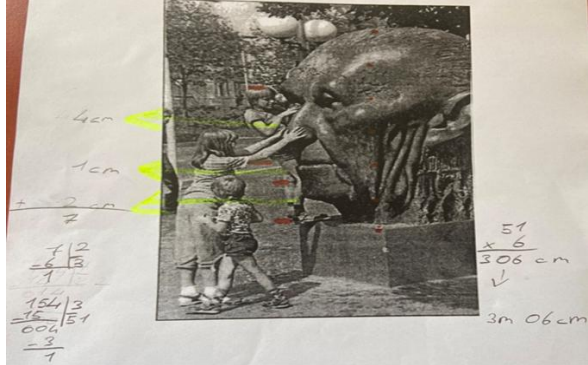
FINDINGS

The developmental trajectories of the three focus participants, Snapdragon (low), Lemur (medium), and Dolphin (high), were holistically assessed across four sequential clinical interviews. The following findings illustrate the progression of their cognitive mathematical modeling competencies, from the baseline established in the "Why BIG-S burger?" activity to the final evaluation in the "Adenauer Statue" activity. The findings are presented sequentially to highlight the shifts in each student's modeling proficiency throughout the intervention.

Snapdragon's Development of Cognitive Mathematical Modeling Competence

The cognitive mathematical modeling competence levels of Snapdragon, a low-level student, in model-eliciting activities during clinical interviews and the scores he received in line with these levels are given in **Figure 2**. The indicators for each mathematical modeling process of Snapdragon are presented in **Table 7**.

Table 7. Mathematical modeling process of Snapdragon

Competency	Score	Clinical interview	Citations/indications
Understanding the task	2	Pre-clinical interview	Snapdragon: <i>Why is there a BIG-S burger variety? The employee asks why they choose this one more, why they want this one. In other words, the restaurant manager. He wants us to find it.</i> Researcher/teacher: <i>What will you find?</i> Snapdragon: <i>Why they want the BIG-S burger so much. That's how I understand it.</i> Researcher/teacher: <i>How will you solve it?</i> Snapdragon: <i>I'll look at this table right now, teacher. Then I'll decide. My teacher says the most important here, in other words, he wants us to rank the most popular variety from the most disliked. We'll explain this with a letter. Now I'll look at the results of this survey.</i>
Simplifying	3	Second mid-clinical interview	Researcher/teacher: <i>How will you solve the problem?</i> Snapdragon: <i>Teacher, I gave 9 cities here. 9 is the highest score, 1 is the lowest score. I will rank them that way and then group them.</i> Researcher/teacher: <i>Which ones will you consider, Snapdragon?</i> Snapdragon: <i>They want the weather to be nice but not too sunny.</i> Researcher/teacher: <i>Do they want it not to be too sunny or not too hot? Is it always too hot when the sun is out? For example, right now it is sunny but hot?</i> Snapdragon: <i>No, teacher. Then I will give the highest to the one with the sunniest days.</i> Researcher/teacher: <i>Is the number of days over 30°C too hot?</i> Snapdragon: <i>It is hot, teacher. Then the highest score is Eskişehir.</i> Researcher/teacher: <i>What about the annual average rainfall?</i> Snapdragon: <i>My teacher is the lowest one, namely Izmir. Because it will be held outdoors, there should be little rain.</i>
Mathematizing	1	First mid-clinical interview	Snapdragon: <i>There are 24 hours in a day. Let's say he sleeps for 10 hours. That leaves 14 hours. My teacher works for 6 hours. We will subtract 6 from 14. 8. He has 8 hours of free time. He sleeps for 10 hours out of 24 hours, which leaves 14 hours. He works for 6 hours, 8 hours of free time. It happens when he works for 6 hours. I think he spends more time at home, that is, outside of work.</i>
Working mathematically	4	Final clinical interview	 The image shows handwritten mathematical work on the left and a photograph of a person working on the right. The handwriting includes a diagram with a horizontal line and points labeled '4cm', '1cm', and '2cm'. Below this is a long division problem: $\begin{array}{r} 12 \\ 13 \overline{) 154} \\ \underline{156} \\ 154 \\ \underline{156} \\ 00 \\ 00 \\ \underline{00} \\ 00 \end{array}$ To the right of the division is a multiplication problem: $\begin{array}{r} 51 \\ \times 6 \\ \hline 306 \end{array}$ Below the multiplication is the result '3m 06cm'. The photograph shows a person working on a large, dark, textured object, possibly a sculpture or a large piece of machinery.
Interpreting	1	First mid-clinical interview	Mektup: Beyza'nın annesi matbaada 6 saat çalışıyor. Bir günün 10 saatini uyuyorsa geriye 14 saat kalır. 14 saatin 6 saatini çalışıyor, 8 saatini boş zamanında kullanıyor. Yani iş dışında daha çok zaman geçiriyor. Snapdragon: <i>Beyza's mother works 6 hours a day at the printing house. If she sleeps for 10 hours a day, there are 14 hours left. She works for 6 hours out of 14 hours and uses 8 hours for her free time. In other words, she spends more time outside of work.</i>
Validating	4	First mid-clinical interview	Snapdragon: <i>I'll check, teacher (he's checking his work).</i>

Understanding the task

As seen in **Figure 2**, in the pre-clinical interview, it was observed that the student included statements indicating that he understood the problem, determined what was given and what was requested, but could not establish a relationship between them. In the first mid-clinical interview, it was observed that the student included statements indicating that he understood the problem to some extent, determined what was given

and what was requested to some extent. In the second mid-clinical interview, it was determined that the student included statements indicating that the problem was fully understood but made minor mistakes while determining what was given and what was requested and was able to establish a relationship between them. In the final clinical interview, it was determined that the student included statements indicating that the problem was fully understood, determined what was given and what was requested, and was able to establish a relationship between them.

Simplifying

As seen in **Figure 2**, in the pre-clinical interview, the student did not determine what was given and what was requested and directly entered the world of mathematics. In other words, he did not make any realistic assumptions about how to solve the problem and tended to solve the problem directly. In the first mid-clinical interview, it was determined that the student simplified the problem to some extent and made acceptable assumptions within this scope to some extent. In the second mid-clinical interview, it was determined that the student simplified the problem, determined the necessary/unnecessary variables and made realistic assumptions. In the final clinical interview, it was also seen that the student was able to make realistic assumptions.

Mathematizing and working mathematically

As seen in **Figure 2**, it was observed that the student created the necessary mathematical models correctly and solved the created mathematical models correctly in the pre-clinical interview. It was observed that the student created incomplete mathematical models and solved the created mathematical models correctly in the first mid-clinical interview. In the second mid-clinical and final clinical interview, the student created correct mathematical models according to realistic assumptions and solved the created mathematical models correctly.

Interpreting

Figure 2 indicates that the student failed to demonstrate interpretation competence regarding the mathematical solution obtained in the pre-clinical interview. In the first mid-clinical interview, the student interpreted the incomplete mathematical solution he obtained in a deficient manner in the real-life context. In the second mid-clinical and final clinical interviews, the student interpreted the correct mathematical solution he obtained in a deficient manner in the real-life context.

Validating

Figure 2 indicates that the student failed to demonstrate validation competence for the mathematical solution in the pre-clinical interview. The student demonstrated validating competence for the first time in the first mid-clinical interview. In this process, he only checked the mathematical operations he performed but did not correct these errors despite his mistakes. In the second mid-clinical and final clinical interviews, the student was found to have validating competence because he checked the mathematical operations he performed during the problem-solving process.

Lemur's Development of Cognitive Mathematical Modeling Competence

The levels of cognitive mathematical modeling competencies of Lemur, an intermediate level student, in model-eliciting activities during the clinical interviews and the scores he received in line with these levels are given in **Figure 3**. The indicators for each mathematical modeling process of Lemur are presented in **Table 8**.

Understanding the task

As seen in **Figure 3**, Lemur demonstrated a gradual improvement in understanding the task across the clinical interviews. In the pre-clinical interview, he was able to identify the information given and what was requested but did not establish a relationship between them. In the first mid-clinical interview, his identification of the givens and goals became less consistent, indicating partial understanding of the task. By the second mid-clinical interview, Lemur demonstrated complete understanding, identified the givens and goals, and established a relationship between them. This level of performance was maintained in the final

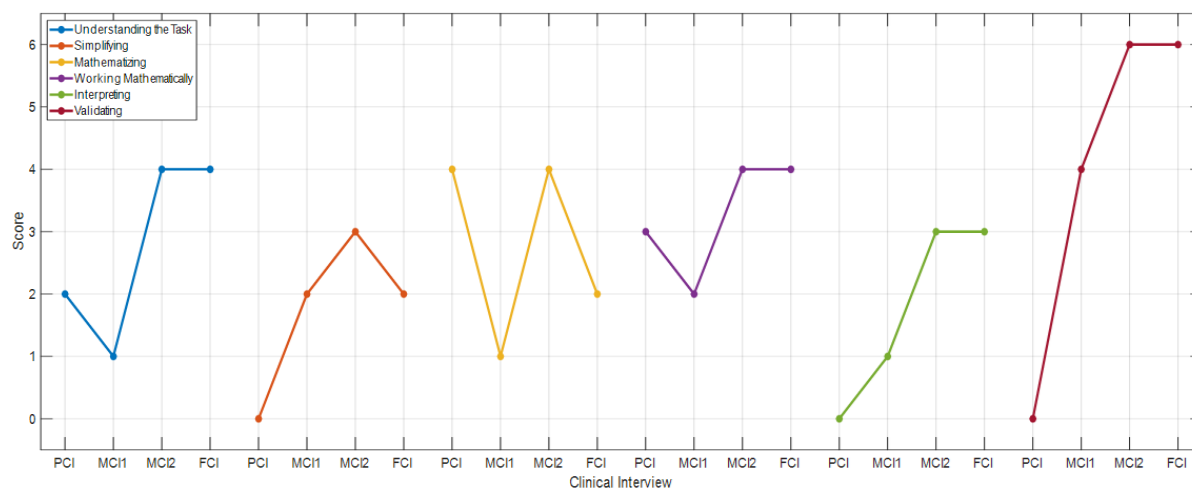


Figure 3. Lemur's development of cognitive mathematical modeling competence (PCI: Pre-clinical interview; MCI1: First mid-clinical interview; MCI2: Second mid-clinical interview; FCI: Final clinical interview) (the authors' own elaboration)

clinical interview. Thus, Lemur's development was characterized by a transition from identifying the components of a problem without relating them to one another to establishing meaningful relationships among the givens and goals.

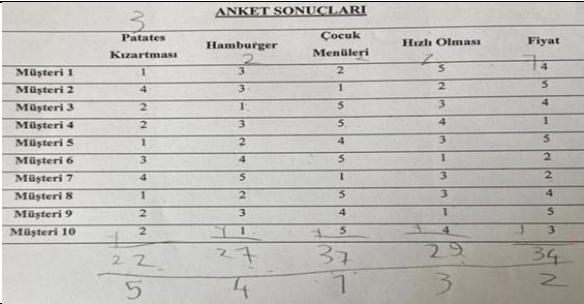
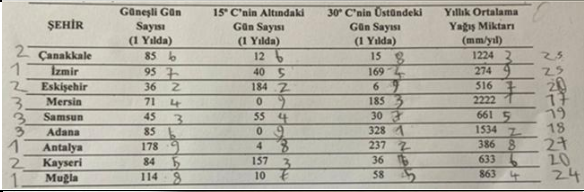
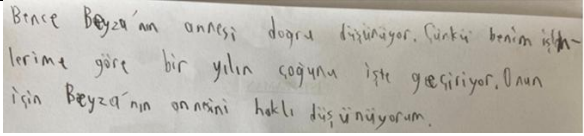
Simplifying

As shown in **Figure 3**, Lemur initially tended to move directly to mathematical operations without identifying the given and requested information or making realistic assumptions. In the first mid-clinical interview, he began to simplify the problem and make partially acceptable assumptions. By the second mid-clinical interview, he demonstrated a more developed simplifying process by identifying necessary and unnecessary variables and making realistic assumptions. However, this level of performance was not fully

Table 8. Mathematical modeling process of Lemur

Competency	Score	Clinical interview	Citations/indications
Understanding the task	1	First mid-clinical interview	Researcher/teacher: <i>What did you understand from the problem?</i> Lemur: <i>Beyza's mother thinks that she spends too much time at work. She thinks that time passes too quickly at work. What she wants from us is to find out whether Beyza's mother is right and to write a solution in a letter?</i> Researcher/teacher: <i>What is she right about?</i> Lemur: <i>She thinks that time passes too quickly. What she wants from us is to find out whether Beyza's mother's thought is correct.</i>
Simplifying	2	Final clinical interview	Lemur: <i>I thought it would give a more accurate result since it is on a statue.</i> Researcher/teacher: <i>Okay. How is this child's posture? Is it straight?</i> Lemur: <i>It doesn't lie completely straight. At most, up to here (showing the part from his foot to his knee). It is crooked from here (showing the part after the knee) (measuring the part from his foot to his knee with a ruler). 2 cm. But this is according to us, according to the picture. It can't be 2 cm in reality (measuring from the knee to the waist with a ruler). 3 cm (measuring from the waist to the head with a ruler). 2 cm.</i> Researcher/teacher: <i>How many cm is the child in the image then?</i> Lemur: <i>It is 7 cm. But according to this picture. We don't know how much it was reduced. If my height is 140 cm, his height could be the same. If I compare it to myself ... I think I am taller than that, but I don't know. The child may be 9 years old. 1 min ... (thinking) I thought of something. Now my height is 140. I am 10 years old. We will find that approximately. Let's say he is my age. Let's say he is 140 cm. We will find this place (the remaining part of the statue after measuring the child's height) (measuring the remaining part of the statue with a ruler). 3 cm.</i> Researcher/teacher: <i>You found the child's height as 7 cm. But you assumed that he was actually 140 cm. What kind of a relationship could there be between these?</i> Lemur: <i>20 cm, 1 piece, that is, 1 piece is 20 cm.</i>

Table 8 (Continued).

Competency	Score	Clinical interview	Citations/indications
Mathematizing	4	Pre-clinical interview	
Working mathematically	4	Second mid-clinical interview	
Interpreting	1	First mid-clinical interview	 Lemur: <i>I think Beyza's mother is right. Because compared to my job, she spends most of the year at work. That's why I think Beyza's mother is right.</i>
Validating	4	First mid-clinical interview	Lemur: <i>Just in case, let me check (student checks his mathematical operations). I think it's over, teacher.</i>

maintained in the final clinical interview, when his assumptions were again only partially acceptable. Thus, Lemur showed substantial development in simplifying during the intervention.

Mathematizing and working mathematically

As seen in Figure 3, it was observed that the student created the necessary mathematical models correctly in the pre-clinical interview but made mistakes in solving the mathematical model. In the first mid-clinical interview, it was observed that the student created incomplete mathematical models and solved the mathematical models correctly. In the second mid-clinical interview, the student created correct mathematical models based on realistic assumptions and solved the mathematical models correctly. In the final clinical interview, the student created correct mathematical models based on somewhat acceptable assumptions and solved the mathematical models correctly.

Interpreting

Figure 3 indicates that the student failed to demonstrate interpretation competence for the mathematical solution in the pre-clinical interview. In the first mid-clinical interview, the student interpreted the incomplete mathematical solution he obtained in a real-life context incompletely. In the second mid-clinical and final clinical interviews, the student interpreted the correct mathematical solution he obtained in a real-life context incompletely.

Validating

As illustrated in Figure 3, it was determined that the student did not exhibit validation competence with respect to the mathematical solution obtained in the pre-clinical interview. The student demonstrated validating competence for the first time in the first mid-clinical interview. In this process, he only checked the mathematical operations he did, but he did not correct these errors even though he had made mistakes. In the second mid-clinical and final clinical interviews, the student was found to have validating competence because he checked the mathematical operations he made both during and after the problem-solving process.

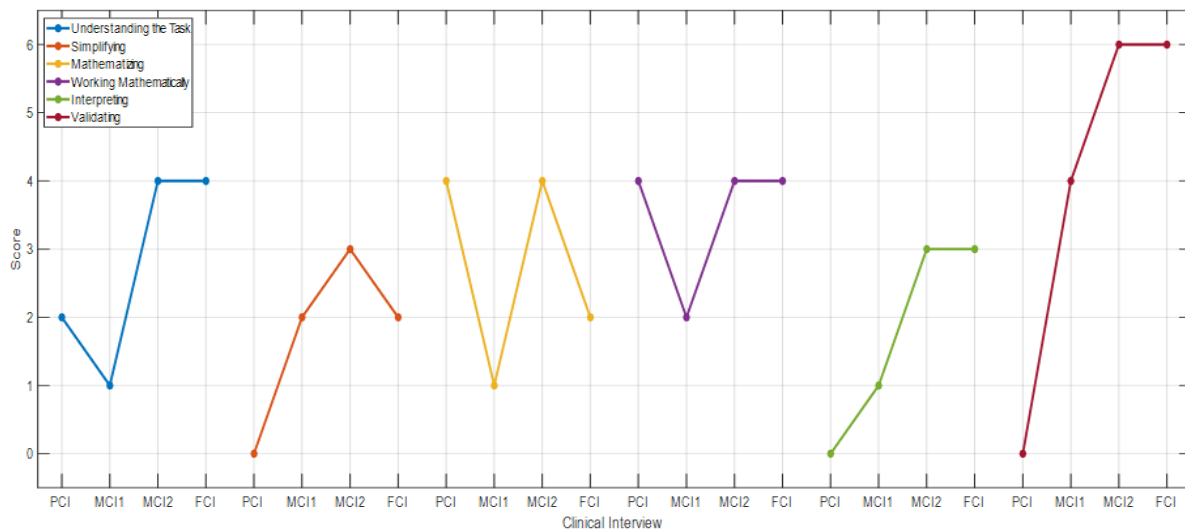


Figure 4. Dolphin's development of cognitive mathematical modeling competence (PCI: Pre-clinical interview; MCI1: First mid-clinical interview; MCI2: Second mid-clinical interview; FCI: Final clinical interview) (the authors' own elaboration)

Dolphin's Development of Cognitive Mathematical Modeling Competence

The levels of cognitive mathematical modeling competencies of Dolphin, a high-level student, in model-eliciting activities during clinical interviews and the scores he received in line with these levels are given in **Figure 4**. The indicators for each mathematical modeling process of Dolphin are presented in **Table 9**.

Understanding the task

As seen in **Figure 4**, Dolphin initially demonstrated an understanding of the problem and was able to identify the givens and goals, but he did not establish a relationship between them. In the first mid-clinical interview, his performance reflected a more partial understanding of the task. By the second mid-clinical

Table 9. Mathematical modeling process of Dolphin

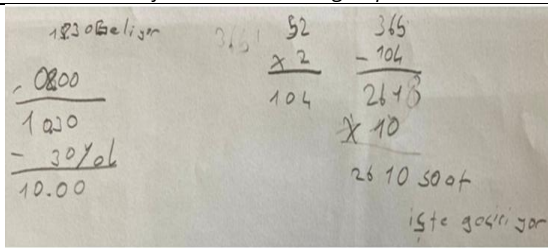
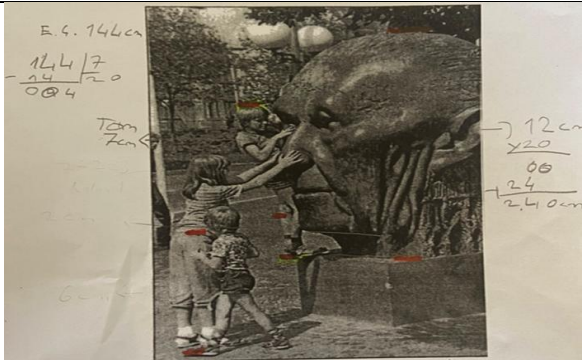
Competency	Score	Clinical interview	Citations/indications
Understanding the task	2	Pre-clinical interview	Dolphin: My teacher, there was a restaurant called BIG-S burger. The manager of the restaurant asked why they preferred BIG-S burger. And he wanted to find 5 different proofs of this. He conducted a survey with 10 people. People would rate it based on its fries, hamburger, kids menu, speed and price. 1 point was the lowest and 5 points was the highest. Then he asked me to rank it from most important to least important.
Simplifying	3	Second mid-clinical interview	Researcher/teacher: So, do you think the number of days above 30°C is hot or warm? Dolphin: It is hot, teacher. Researcher/teacher: So, which criteria will you take into consideration? Dolphin: All of them, teacher. I will especially take this into consideration in the scoring. If the number of days below 15 °C is 0, I will give a higher score. Because 15°C is low. Researcher/teacher: Is the annual average rainfall important? Dolphin: I think it is very important. Because they want to do it outdoors. Researcher/teacher: Who will you give the highest score for the number of days below 15 °C? Dolphin: The lowest. I will score first, then add and group.
Mathematizing	1	First mid-clinical interview	

Table 9 (Continued).

Competency	Score	Clinical interview	Citations/indications
Working mathematically	4	Final clinical interview	 The image shows two parts: on the left, a piece of paper with handwritten calculations. At the top, it says 'E.S. 144 cm'. Below that is a long division problem: $144 \overline{) 2000}$. The quotient '138' is written, and the remainder '8' is shown. Below the division, it says 'Tom Pen'. On the right, there is a photograph of a student sitting at a desk, working on a problem. To the right of the photograph, there are handwritten calculations: 12 cm, 20, 08, 24, and $2,40 \text{ cm}$.
Interpreting	3	Second mid-clinical interview	Dolphin: <i>I took all the details into consideration and I scored them as we did in the previous problems and added up the points. Then I listed them here as the best cities, good cities and bad cities. And I wrote a letter here that if you want to see the score, you can look at it here.</i>
Validating	4	First mid-clinical interview	Dolphin: <i>After we finished our solution, we were checking it (he reads the problem again, checks the operations he did). I solved it correctly.</i>

interview, however, Dolphin demonstrated complete understanding, identified the givens and goals, and established a relationship between them. He maintained his level of performance in the final clinical interview. Thus, despite his relatively high initial overall modeling competency, Dolphin showed a clear developmental shift in this specific competency, particularly in establishing relationships among the givens and goals.

Simplifying

As shown in Figure 4, Dolphin initially tended to move directly to mathematical operations without identifying the given and requested information or making realistic assumptions. Similar to Lemur, he began to simplify the problem and make partially acceptable assumptions in the first mid-clinical interview. By the second mid-clinical interview, Dolphin was able to identify necessary and unnecessary variables and make realistic assumptions. In the final clinical interview, however, his assumptions were again only partially acceptable. Thus, Dolphin followed a developmental pattern similar to Lemur's, characterized by improvement in simplifying followed by some fluctuation at the final stage.

Mathematizing and working mathematically

As seen in Figure 4, it was observed that the student created the necessary mathematical models correctly and solved the created mathematical models correctly in the pre-clinical interview. In the first mid-clinical interview, it was observed that the student created incomplete mathematical models and solved the created mathematical models correctly. In the second mid-clinical interview, the student created correct mathematical models based on realistic assumptions and solved the created mathematical models correctly. In the final clinical interview, the student created correct mathematical models based on somewhat acceptable assumptions and solved the created mathematical models correctly.

Interpreting

As shown in Figure 4, the student was found not to demonstrate interpretation competence for the mathematical solution in the pre-clinical interview. In the first mid-clinical interview, the student interpreted the incomplete mathematical solution he obtained in a deficient manner in the real-life context. In the second mid-clinical and final clinical interviews, the student interpreted the correct mathematical solution he obtained in a deficient manner in the real-life context.

Validating

As shown in Figure 4, the student was found not to demonstrate validation competence for the mathematical solution in the pre-clinical interview. The student demonstrated validating competence for the

first time in the first mid-clinical interview. In this process, he tried to understand the problem by rereading it and checked the mathematical operations he performed, so he adopted a validating approach. However, despite creating incomplete mathematical models, he could not notice the errors he made. Therefore, he adopted a validating approach, and although the errors were determined, he did not correct them. In the second mid-clinical interview, the student adopted a validating approach, since he checked the mathematical operations he performed during the problem-solving process and corrected the errors he noticed. In the last clinical interview, the student was found to have validating competence, since he checked the mathematical operations he performed both during and after the problem-solving process.

DISCUSSION

This study explored the impact of cooperative learning-integrated model-eliciting activities on 4th-grade students' cognitive mathematical modeling competencies. Consistent with Borromeo Ferri's (2006) cognitive perspective, initial observations during the pre- and first mid-clinical interviews revealed that students struggled significantly with understanding the task. Instead of identifying given information, students exhibited a tendency to transition directly into mathematical operations into common behavior documented by Indahyanti (2017), where students bypass sense-making for immediate solution-seeking. However, by the second mid-clinical interview, students demonstrated a marked improvement, successfully articulating problem constraints in their own words. This shift suggests that the integrated cooperative learning strategies, specifically the individual "thinking" stages, played a crucial role. By requiring students to reflect individually before group interaction, these strategies countered the impulsive "direct-solve" habit, fostering deeper problem comprehension (Astuti, 2014). Furthermore, the developmental progress observed throughout the teaching experiment supports the notion that modeling competence is an experiential skill. While initial difficulties can be attributed to students' lack of exposure to model-eliciting activities in traditional curricula (Haas et al., 2020), sustained engagement led to enhanced proficiency, aligning with findings that modeling skills mature as students gain experience (Canbazoglu-Albayrak & Tarım, 2025; Leiss et al., 2010). These findings suggest that early opportunities for individual reflection within cooperative learning environments may facilitate the transition from procedural engagement to conceptual understanding during the initial stages of mathematical modeling.

Similar to the initial struggle with task understanding, students faced significant challenges with problem simplification during the pre and first mid-clinical interviews. During this phase, participants struggled to distinguish between relevant and irrelevant variables and failed to generate realistic assumptions a difficulty widely reported across various age groups (Blum, 2015; Chan et al., 2012; Maaß, 2006). This direct-solved orientation often led to incomplete or unrealistic mental models. A pivotal shift occurred by the second mid-clinical interview, where students began systematically identifying variables and constructing viable assumptions. This progress can be attributed to the "talking and discussion" stages of the implemented cooperative strategies. These interactive phases forced students to externalize their internal assumptions, allowing peer feedback to fill the gaps in their reasoning. As Maaß (2006) notes, individual problem-solving often results in fragmented assumptions; however, the social nature of cooperative learning facilitates the essential "filtering" process deciding which real-world information to include or exclude (Mousoulides et al., 2008). Since the ability to simplify a real-life situation and organize the resulting data set is a prerequisite for successful mathematizing (English, 2006), the development of this competence served as a cognitive bridge for the subsequent stages of the modeling cycle in this study. This finding highlight that simplifying should not be viewed solely as an individual cognitive activity but also as a socially negotiated process in which assumptions are refined through peer interaction.

Throughout the intervention, students generally demonstrated baseline ability for mathematizing and working mathematically, though their initial approaches were result-oriented and lacked contextual depth. In the pre and first mid-clinical interviews, students exhibited a strong desire for immediate solutions (Greefrath, 2015), often performing mental arithmetic with numerical data regardless of its relevance to the problem context. This tendency to force numbers into pre-existing, unsuitable schemes aligns with Blum's (2011) findings on how students bypass meaningful modeling for familiar procedural routines. During this early phase, mathematical models were often subjective or intuitive rather than systematic (Doerr & English, 2003).

A significant progression was observed from the second mid-clinical interview onward, as students' verbal expressions evolved and their models began to align with realistic assumptions. This developmental shift mirrors Borromeo Ferri's (2006) observations that modeling proficiency matures through sustained practice. A key catalyst for this improvement was the "writing" stage of the implemented cooperative strategies. By requiring students to document their reasoning, these strategies facilitated the externalization of mental constructs into visual and symbolic representations (Pratiwi, 2019). This transition from mental calculation to formal modeling underscores the importance of structured reflection in bridging the gap between real-world contexts and mathematical operations. This finding suggests that opportunities to externalize reasoning through writing may support students in coordinating real-world situations with mathematical representations.

The findings indicate that interpreting was the stage where students encountered the greatest difficulty. Participants often struggled to translate their mathematical results back into real-life contexts, at times completely refraining from such interpretations. This hesitation likely stems from traditional learning environments that prioritize reaching a "correct" numerical answer over making sense of that answer in non-mathematical terms (Haas et al., 2020; Lesh & Doerr, 2003). The "talking-discussion" stage of the cooperative strategies proved essential in mitigating this challenge. By discussing their findings within a group setting, students were compelled to evaluate whether their mathematical solutions remained meaningful in the real world. This social interaction provided the necessary environment for students to interpret their models and assess their practical accuracy, reinforcing the necessity of providing spaces where students can bridge the gap between abstract results and lived reality (Blum & Borromeo Ferri, 2009; Suh et al., 2021). The findings therefore emphasize that interpretation develops not only through mathematical competence but also through opportunities to justify and negotiate meaning with peers.

Regarding validating competence, students initially exhibited a narrow focus, typically limiting their verification to checking for operational or calculation errors. During the early interventions, they failed to assess the overall accuracy of their assumptions or the viability of their mathematical models. This finding aligns with observations by Blum (2011), Borromeo Ferri (2006), and Maaß (2006), who noted that novice modelers often equate validation with mere procedural accuracy rather than holistic model evaluation. The "presentation" phase of the implemented cooperative strategies proved instrumental in expanding this competence. By requiring students to prepare their solutions for peer review, the strategy created a natural incentive for them to re-examine their work and identify conceptual flaws before going public. This process moved beyond simple arithmetic checking; it engaged higher-order thinking skills, allowing students to reason through their logic individually and refine it through group discussion (Sugiarto & Sumarsono, 2014). Consequently, the social pressure and structured dialogue of the presentation phase acted as a catalyst for students to transition from procedural checking to genuine model validation. These results suggest that validation should be conceptualized as a reflective and collaborative process rather than merely an individual checking procedure.

An important finding concerns the developmental trajectories of the three focal students who entered the intervention at different levels of modeling proficiency. Despite their different initial RAMS scores, Snapdragon (low), Lemur (medium), and Dolphin (high), all three students demonstrated development across the six cognitive mathematical modeling competencies. However, their trajectories were not identical or strictly linear. Snapdragon, who began at the lowest proficiency level, showed substantial development across the modeling process, while Lemur and Dolphin, despite their higher initial proficiency levels, also demonstrated clear development across the six competencies. In several competencies, Lemur and Dolphin showed similar developmental patterns, whereas Snapdragon sometimes reached comparable levels through a different trajectory. Notably, all three students were reported to have "good" general mathematics achievement, despite their substantially different initial modeling competency scores. This discrepancy suggests that general mathematics achievement and cognitive mathematical modeling competency may not develop in parallel and that conventional mathematics achievement may not fully capture students' modeling capabilities. Although the present study did not directly examine the relationship between these two constructs, this pattern highlights the value of considering mathematical modeling as a distinct dimension of mathematical competence. Overall, the findings suggest that students with different initial proficiency levels

can develop their cognitive mathematical modeling competencies within a cooperative learning-integrated modeling environment.

CONCLUSION

This study examined the development of 4th-grade students' cognitive mathematical modeling competencies during the implementation of model-eliciting activities within a cooperative learning framework. Overall, the findings indicate that students demonstrated progressive development across the six cognitive mathematical modeling competencies throughout the teaching experiment. Rather than suggesting that cooperative learning alone accounts for this development, the findings indicate that structured opportunities for individual reflection, peer discussion, collaborative reasoning, and written justification can support students' engagement with different stages of the mathematical modeling process.

From a theoretical perspective, this study contributes to the mathematical modeling literature by extending the cognitive perspective of Borromeo Ferri (2006) to a collaborative learning context. While cognitive mathematical modeling competencies are commonly conceptualized as individual reasoning processes, the present findings suggest that their development may also be supported through structured peer interaction. In particular, the findings highlight that different cooperative learning phases (thinking, discussing, writing, and presenting) may provide complementary opportunities for students to develop competencies related to understanding the task, simplifying, mathematizing, working mathematically, interpreting, and validating. Thus, the study offers a more integrated perspective on how cognitive and social processes interact during elementary mathematical modeling.

The findings also contribute to cooperative learning research by illustrating how cooperative learning strategies can function not merely as classroom management techniques but as pedagogical structures that support mathematical reasoning during complex problem-solving processes. Rather than emphasizing collaboration solely as a means of increasing participation, this study demonstrates how structured interaction protocols such as think-talk-write, think-pair-share, and timed pair-share may facilitate students' reflection, justification, and refinement of mathematical ideas throughout the modeling cycle.

From an educational perspective, the findings have important implications for elementary mathematics classrooms. The results suggest that model-eliciting activities implemented within cooperative learning environments can provide meaningful opportunities for young learners to discuss assumptions, justify mathematical decisions, interpret solutions in real-world contexts, and evaluate the appropriateness of their models. Accordingly, mathematics instruction should provide students with opportunities not only to obtain correct numerical answers but also to explain, revise, and validate their mathematical thinking through structured collaborative experiences.

Limitations and Suggestions

Despite its contributions, this study has several limitations that provide avenues for future research. First, the small sample size ($n = 3$ for focus participants) is inherent to the qualitative nature of a teaching experiment aimed at high-resolution cognitive tracking; however, future studies could involve larger or more diverse student populations to enhance the generalizability of the findings. Second, the assessment was bounded by the specific model-eliciting activities and the RAMS instrument utilized. Subsequent research could design diverse MEAs for the elementary level and employ alternative assessment frameworks to capture different dimensions of modeling proficiency. Finally, students' competencies were evaluated specifically through the lens of Borromeo Ferri's (2006) modeling cycle. While this provided a robust cognitive framework, future studies might explore students' development using alternative modeling perspectives (e.g., realistic or socio-critical modeling) to offer a broader understanding of the modeling process in early childhood education.

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