



History of mathematics in teaching: Students' solutions to Fibonacci's problem "A lion, a leopard and a bear"

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ABSTRACT

This paper presents an initial exploratory analysis of the use of historical problems in mathematics education. The problems posed by Fibonacci over 800 years ago, besides offering a glimpse into the past, allow students to connect mathematics to real life and arrive at solutions different from those proposed by Fibonacci, thus enriching the learning experience. Fibonacci's problem "Of a lion, a leopard, and a bear" was given to 113 students in a Mexican high school for solution. The task was structured so that students would follow the four steps proposed by Pólya (1989) for solving a mathematical problem. This paper presents the results of a qualitative analysis of the students' performance in planning their solutions. Ten students found the Fibonacci's solution. Sixteen students, using different modelling, got different solutions that might be considered correct. These students' responses reveal not only their knowledge of mathematics and its procedures but also their understanding of the limitations of school mathematics problems when applied to real-world situations.

Keywords: problem-solving, historical problems, realistic considerations, Fibonacci

INTRODUCTION

The use of historical problems in teaching school mathematics exposes students to situations that enrich learning, as it provides a glimpse "behind the concepts and theories taught in school, as well as their relationship to other school subjects and to real life" (Furinghetti, 2020, p. 989). The book "*Liber abaci*", written by Fibonacci in 1202, contains a series of problems, especially in chapter 12, which have been used throughout the years in different contexts and with different formulations, making them challenging mathematical puzzles. This paper analyses the answers given by high school students to Fibonacci's historical problem "Of a lion, a leopard, and a bear." The students' responses provide information about how they connect reality with the classroom context, assuming that there are different ways to solve a problem, and that, depending on the modelling they follow in their procedures, the results can also be different.

The planning and procedures described by the students to arrive at the solution allow us to analyze and understand that some of the answers, different from the result given by Fibonacci, far from being incorrect, demonstrate the students' use of their real-world knowledge to arrive at an answer. Only by reading their arguments can their answers be validated and the modelling understood.

The results presented in this article are part of Claudia Éthel Figueroa Suárez's PhD thesis.

THEORETICAL FRAMEWORK

Problem-Solving and Learning Environments

Problem-solving is considered an effective way to learn school mathematics. A student who knows how to solve a problem does so because they can and know how to apply what they have learned (Corts & Vega, 2004; De Corte et al., 2000). Schoenfeld (1992, p. 348, 2016) distinguishes five important aptitudes for problem-solving:

- “basic knowledge,
- problem-solving strategies (heuristics),
- monitoring and control (self-regulation),
- beliefs and affect (students, teachers, and society), and
- practice.”

De Corte (2004, p. 282) concludes that five aptitude categories have been identified that students should acquire to develop a good aptitude for mathematics:

- “good organization and accessibility of basic mathematical knowledge,
- heuristic methods, problem-solving skills,
- metacognition, knowledge of cognitive functions,
- self-regulation techniques, and
- beliefs about oneself in relation to mathematics and problem-solving, about the classroom context, and about mathematics and its learning.”

These categories of aptitudes, both for problem-solving and for willingness to learn mathematics, are related to the principles that should guide learning environments (De Corte, 1995; Santos-Trigo, 2024). The connection between the development of these aptitudes and the design of learning environments lies in the instruction and the way in which a student is asked to solve a problem. Pólya (1989) analyzed the processes of those who solve mathematical problems. Pólya (1989) proposed, in his book *“How to solve it”*, the following steps for solving a problem:

- accept and understand the conditions of the problem,
- plan your solution,
- carry out the planned solution, and
- check the solution.

These four steps provide a general process for problem-solving. Students must think of a path to solve a problem, in contrast to algorithms, which are well-defined processes (Kilpatrick et al., 1998). In this way, students can explore various ways to solve a problem without resorting to a set procedure and arrive at a solution using their own resources. They participate actively, critically, and creatively in their learning, rather than as passive recipients.

This also allows teachers to monitor the solution process at all its stages. These different stages, when recorded in writing, not only help students follow a clear and orderly procedure but also provide a record so that teachers can better understand how they arrive at a result. The idea is that students follow these steps to solve a problem, and teachers can read and analyze the solution process, not just the answer.

The Use of History in Mathematics Education

Incorporating epistemological and historical elements of mathematics into mathematics education not only enriches knowledge about this discipline but also reveals mathematics as the result of “a living human intellectual endeavor with a long history and a vibrant present” (Chorlay et al., 2022, p. 1409). A historical approach allows us to connect mathematics with other intellectual and cultural activities, revealing the scientific, social, and economic conditions under which mathematical knowledge developed. As a result of this interaction, we understand that learning mathematics is not just about becoming competent in the handling, knowledge, manipulation, and use of theories and tools. It is about finding relationships between these

concepts and making sense of them through the construction and meanings that have been given over time (Chorlay et al., 2022).

According to Jankvist (2009), the use of history in mathematics education can help students overcome difficulties by employing teaching situations that exemplify these difficulties and provide an analysis of the nature of knowledge and the historical ideas that generated it. History is also used to “maintain students’ interest and enthusiasm in the subject” (Jankvist, 2009, p. 237). Fauvel (1991) points out that the use of history provides the human facet of mathematics. Following this same line of reasoning, Liu (2003) highlights that if students are given the opportunity to learn about the challenges faced by mathematics and mathematicians throughout history—that is, if they are shown the human side of this science—it can break down their perception of mathematics as inaccessible, creating a barrier to learning.

PREVIOUS EXPERIMENTAL STUDIES

The Use of Historical Problems in Teaching

There are different ways to introduce history into mathematics teaching. The approach taken depends on the teaching and research objectives (Clark et al., 2016; Furinghetti, 2020; Jankvist, 2009; Tzanakis et al., 2002; Tzanakis & Thomaidis, 2011). One way to introduce history into teaching is using historical problems, which appear in different contexts and cultures throughout history (Moyon, 2019). Historical problems are the same ones that have helped in the development of thought processes and that motivated and gave rise to mathematical problems and the reflections that led to the construction of knowledge (Liu, 2003).

In his book *“Mathematical expeditions: Exploring word problems through the ages”*, Swetz (2012) takes a journey through history and around the world, compiling problems that illustrate the development of mathematics throughout history and showcasing mathematical knowledge in different cultures. This intercultural approach to mathematics is highly useful for teachers who want to present mathematics to their students from a historical and cultural perspective. Swetz (2012) also explains how these problems, which he called words problems, were used for teaching. The authors carefully selected the mathematical knowledge they wanted to communicate and encouraged the reader to want to solve them by presenting activities that motivated them socially and/or intellectually.

Leonardo of Pisa, better known as Fibonacci, introduced Arabic numerals to Europe in his book *“Liber abaci”* and presented a series of problems. Fibonacci was born into a merchant family on the Tuscan Coast, which allowed him to visit various regions of the Islamic world, bringing with him the mathematical knowledge they possessed through trade. Fibonacci’s problems, in addition to their intercultural nature, offer the possibility of using different approaches to the solution and have already been used in mathematics education research (Gil & Martinho, 2016; Juárez Ramírez et al., 2014; Moyon, 2019; Sanz & Gómez, 2018; Silisko, 2020; Taşkın et al., 2013). Furthermore, different contemporary solution strategies can be compared with those presented by Fibonacci, as well as incorrect solutions. In other words, they are an invaluable resource for teaching, as they showcase the dynamic and human nature of mathematics, which can enhance its learning.

Several experimental studies report the influence of using historical problems in mathematics teaching. Ben-Chaim et al. (2019) promote creative mathematical thinking and unconventional problem-solving methods using problems they call puzzles. They conducted their study using three problems, which they first gave to pre-service teachers and then to 12-year-old students. According to Ben-Chaim et al. (2019), using this type of problem in teaching improves learning and the classroom atmosphere. Bütüner and Baki (2020) conducted a study with 24 eighth-grade students using activities related to the history of mathematics. Their objective was for students to see that mathematics is not a static but a developing science, thus influencing their beliefs. According to their results, the participants showed enthusiasm and commitment to completing the activities and understood that mathematics arose from the search for solutions to real problems and needs, demonstrating significant changes in the students’ absolutist perceptions.

Demattè and Furinghetti (2022) conducted a study applying 14 problems from ancient treatises to ninth-grade students aged 14 to 15. According to Demattè and Furinghetti (2022), the problems presented linguistic difficulties, despite the transcriptions. However, it was important not to change the units of measurement,

currencies, and cities, nor the archaic, obsolete words. Additionally, some original figures were included, all with the aim of highlighting the antiquity of these problems and immersing the students in a historical context that could generate genuine emotions. In this way, motivational and cultural aspects were considered, in addition to the mathematical ones.

Rizos and Gkrekas (2023) presented an open-ended problem with historical background to 27 university students and evaluated their solution strategies and reactions. They describe an open-ended problem as “non-routine problems that allow for the interpretation of the situations they describe, are susceptible to multiple correct answers, and can be addressed through various problem-solving strategies” (Rizos & Gkrekas, 2023, p. 3). According to Rizos and Gkrekas (2023), these problems can capture students’ attention and supporting experimentation and the development of multiple problem-solving strategies. They concluded that historical and open-ended problems can generate a sense of engagement in students, which will likely help them improve their problem-solving skills. They propose including problems inspired by the history of mathematics in the curriculum.

METHOD

Objectives and Research Questions

The objective is to analyze students’ strategies for solving Fibonacci’s problem “Of a lion, a leopard, and a bear”, to identify characteristics of critical and creative thinking in their problem-solving processes. The specific objectives of this exploratory experimental study were to answer the following research questions:

- How many students did get Fibonacci’s solution to the historical problem “Of a lion, a leopard, and a bear”?
- What are the solutions to the problem that differ from Fibonacci’s one, and on which realistic considerations are they based?

Study Type and Participants

The analysis of the worksheets was conducted using a qualitative methodology, with an exploratory-interpretive approach. Responses were categorized according to the students’ modelling of problem-solving, based on how the sheep was eaten (consumption) and how the sheep as food was divided.

The research population consisted of high school students. Participants in this study were 113 students aged 15 to 18 years from a Mexican high school. It was a convenience sample to which access was readily available.

The Problem and Fibonacci’s Solution

The problem used in this study was taken from Sigler’s (2003) English translation of Fibonacci’s book “*Liber abaci*”. It is found in chapter 12, which is divided into nine parts. Part three contains a series of problems about fractions, one of which is “Of a lion, a leopard, and a bear.” The problem is stated as follows:

A lion ate a sheep in 4 hours; a leopard in 5 hours; and a bear in 6 hours. The question is, if a sheep were thrown between them, in how many hours would they have devoured it? (Sigler, 2003, pp. 279-280).

Fibonacci’s solution is as follows:

Do this: for the four hours in which the lion eats the sheep, put $\frac{1}{4}$; and for the five hours for the leopard, put $\frac{1}{5}$, and for the six hours for the bear, put $\frac{1}{6}$: and since $\frac{1}{6}$, $\frac{1}{5}$, and $\frac{1}{4}$ have a common denominator of 60, assume that they would devour that sheep in 60 hours. And consider then how many sheep the lion would eat in those 60 hours: since it devours one in four hours, it would devour 15 in 60 hours; And the leopard would devour 12 because 12 is one-fifth of 60. Similarly, the bear would devour 10 since 10 is one-sixth of 60. So, in 60 hours they would eat 15, 12, and 10 sheep, that is, 37. Therefore, in 60 hours, they eat 37 sheep; how long does it take for

PROBLEM

Work out your plan mathematically.

Read the problem carefully.

A lion, a leopard, and a bear

A lion ate a sheep in 4 hours, a leopard in 5 hours, and a bear in 6 hours. If the three of them were given a sheep, in how many hours would they eat it together?

Write out your plan for solving the problem.

Include a drawing (or a diagram or a graph) below that would make your plan clearer.



Answer: It will have taken _____ hours.

Verify your answer however you see fit, mathematically or verbally.

Figure 1. Worksheet (the authors' own creation)

them to eat just one sheep? So multiply 1 by 60 and divide by 37, it is $1 \frac{23}{37}$ hours. And in this time, they devoured this sheep (Sigler, 2003, p. 280).

Fibonacci begins with an assumption to solve the problem, which is very common in his solutions. He does use fractions, but he doesn't perform operations with them. The value of 60 is what is currently known as the common denominator of the fractions $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$, but that term wasn't used in 1202. The value of 60 is also the least common multiple of 4, 5, and 6, a term that Fibonacci also didn't use. The important thing is that this number wasn't chosen randomly. Although Fibonacci doesn't explain where he gets it from, it's understandable why he uses it and not another value.

Today Fibonacci's solution is obtained by solving the following equation: $\frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{1}{x} \frac{37}{60} = \frac{1}{x}$, $x = \frac{1}{37/60} = \frac{60}{37} = 1 \frac{23}{37} \approx 1.62$, where x represents the total number of hours the animals would need to eat the sheep together.

The easiest way to understand the equation is to assume how much of the sheep each would eat in one hour: $\frac{1}{4}$ for the lion, $\frac{1}{5}$ for the leopard, and $\frac{1}{6}$ for the bear. We add these fractions together, and the result is $\frac{37}{60}$. This is the fraction of the sheep the animals eat together in one hour. The next step is to find out how long it would take them to eat a whole sheep. This can be solved using a proportion: if the animals eat $\frac{37}{60}$ of the sheep in one hour, they will need $\frac{60}{37}$ hours to eat a whole sheep.

Instrument

The worksheets were designed following the steps proposed by Pólya (1989) for solving a problem. In addition, after formulating their plan, students were asked to create a drawing. Students were then asked to solve the problem individually, following Pólya's (1989) steps. They were given from 30 to 40 minutes.

Some adjustments were made to the problem statement to ensure clarity. However, these changes did not alter the meaning of the problem or its solution. The worksheet with the instructions is shown in **Figure 1**.

RESULTS AND OBSERVATIONS

A total of 113 students participated but 4 of them did not write a response. Therefore, the responses of 109 students are considered in the analysis. The categorization was done according to the type of consumption and distribution:

Table 1. Categorization of the modelling followed by the students

Problem modelling		Answer				Total
		Correct		Incorrect		
		Same as Fibonacci	Different from Fibonacci	Intuitive	Other	
Simultaneous consumption		10	0	28	11	49
Sequential consumption		0	0	0	13	13
Equitable distribution	Sequential consumption	0	12	0	6	18
	Simultaneous consumption	0	4	0	0	4
Other		0	0	0	25	25
Total		10	16	28	55	109

- *Simultaneous consumption.* The animals eat the sheep at the same time and at their own pace, so some eat more than others.
- *Sequential consumption.* The animals eat one after the other, at their own pace, so some eat more than others.
- *Equitable distribution – sequential consumption.* The sheep is divided into three equal parts. The animals do not eat at the same time but are given their share one after the other. The animals eat at their own pace.
- *Equitable distribution – simultaneous consumption.* The sheep is divided into three equal parts. The animals eat the sheep at the same time and at their own pace.

Students' answers that can be classified as correct:

- *Same as Fibonacci.* The animals eat unequal parts of sheep in simultaneous consumption.
- *Different from Fibonacci.* The animals eat equal parts of sheep in simultaneous or sequential consumption.

Students' answers that are incorrect:

- *Intuitive.* A quick answer that involves obtaining an average consumption (Etcheverry & Slisko, 2021; Kahneman, 2011).
- *Others.* Non-intuitive answers that involve other types of errors and different modelling.

The categorization of the solutions according to the modelling, and whether they are correct or incorrect, is shown in **Table 1**.

Now the first research questions How many students did get Fibonacci's solution to the historical problem "Of a lion, a leopard, and a bear?" can be answered.

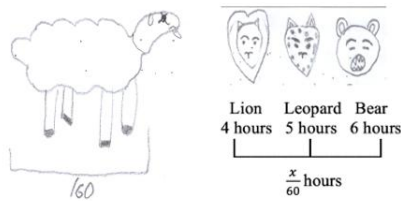
Only 10 of the 109 students arrive at Fibonacci's solution considered mathematically correct. Four students had the same modelling start as Fibonacci supposing that the animals eat unequal parts of the sheep (unequitable distribution). Nevertheless, their answers differ from Fibonacci's one because they make an arithmetic error at the end. Their answers are classified as "other". Although this is not explicitly stated in the procedure, it is assumed that the modelling used by these 14 students to solve the problem involves simultaneous consumption.

Sixteen students, also arriving at a different answer than Fibonacci's, decided to divide the sheep into equal parts, assuming an equitable distribution. They probably don't consider that these three animals could share their food, hence their assumption. These students argued their solution logically and understandably, so although it's not Fibonacci's answer, it might be considered correct. Four students assumed simultaneous consumption, and twelve assumed sequential consumption.

Seventy-nine students did not arrive at the correct solutions. Twenty-eight students followed the simplest and quickest, but incorrect, procedure, which was to average the three feeding times for each animal, arriving at the answer of 5 hours. Another 5 students also obtained averages of 4, 5, and 6 hours, but assumed this was the time it would take to eat 3 sheep, so they again divided the result of 5 hours by 3. Two students correctly explained why they added the fractions $1/4$, $1/5$, and $1/6$, but failed to arrive at the correct answer. The modelling used by these 35 students is considered simultaneous consumption.

Plan: Divide the sheep into fractions so that the rate of consumption, in fractions per hour, can be calculated, then sum these rates to determine the total amount eaten per hour. Once the combined eating rate is known, divide the total number of sheep parts by that rate.

Drawing:



Procedure:

$$\begin{aligned}\frac{60}{4} &= 15 \frac{p}{h} & \frac{60}{5} &= 12 \frac{p}{h} \\ &= 1.621 \text{ hours} & & \\ \frac{60}{6} &= 10 \frac{p}{h} & + &= 37 \frac{p}{h}\end{aligned}$$

Answer: 1.621 hours

Verification: The three animals together would eat the sheep in 1.621 hours or put another way, 97.297 minutes or 5,837.837 seconds.

The key point is to note that they consume 37 parts of the sheep (out of a total of 60) in one hour. To visualize this better: after one hour, they would have already eaten 61.6% of the sheep.

Figure 2. Student 1A's drawing and procedure (the authors' own creation)

Plan: We calculate the rate at which each animal can eat the sheep and add their individual rates together, then find a common denominator to add the fractions, and finally calculate the total time.

Drawing:

Pon abajo un dibujo (o un esquema o una gráfica) que haría más claro tu plan.



Procedure:

$$\begin{aligned}\text{Lion: } &1/4 \\ \text{Leopard: } &1/5 \\ \text{Bear: } &1/6 \\ \frac{1}{4} + \frac{1}{5} + \frac{1}{6} &= \frac{15}{60} + \frac{12}{60} + \frac{10}{60} = \frac{37}{60} \\ \text{Time} &= \frac{60}{37} = 1.62 \text{ hours}\end{aligned}$$

Answer: 1.62 hours

Verification: If you calculate $60/37$, you get approximately 1.62, or you can multiply $37/60$ by 1.62 to verify that the result is 1 (one whole sheep).

Figure 3. Student 2.4D's drawing and procedure (the authors' own creation)

Thirteen students use fractions. Some correctly explain that they represent the consumption rate of each animal, but their solution is the sum of these fractions. The modelling these students follow is considered sequential consumption. Six students also propose dividing the sheep, but this is considered incorrect due to the way they model the problem. Twenty-five students use different procedures and arrive at different incorrect solutions.

The following section shows, in detail and with examples, the categorization of the modelling the students use to arrive at either the correct or incorrect answer.

Correct Solution – Same as Fibonacci's

Simultaneous consumption

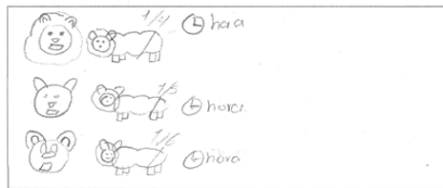
Ten students arrive at the correct solution. They express it as a decimal (1.62 hours) or convert it to minutes (1 hour 37 minutes). One student follows a procedure very similar to Fibonacci's, although without explaining why she or he divides the sheep into 60 parts. This student has a clear understanding of the consumption rate, so her or his procedure is clear and correct (see [Figure 2](#)).

Five of the students who gave the answer of 1.62 hours followed similar procedures. None clearly explained how they arrived at the result. After adding the fractions, which represent the rate of consumption, they did not explain why their result was $60/37$ and not $37/60$, which is the result of the sum. The result of $37/60$ is the fraction of a sheep that the three animals eat in one hour, eating at the same time. Therefore, the time it takes them to eat a whole sheep is $60/37$ (see [Figure 3](#)).

Plan: Divide the sheep into groups based on feeding time, as each animal eats at a different speed.

Drawing:

Pon abajo un dibujo (o un esquema o una gráfica) que haría más claro tu plan.



Procedure:

$$\frac{1}{4} + \frac{1}{5} + \frac{1}{6}$$

$$1/4 = 15/60$$

$$1/5 = 12/60$$

$$1/6 = 10/60$$

$$37/60$$

$$1 + \left(\frac{37}{60}\right) = \frac{60}{37} = 1:37$$

Answer: 1:37 hours

Verification: Dividing the sheep into fractions makes the calculation easier because each animal eats at a different speed; summing all the time gives the result in hours.

Figure 4. Student 6.1D's drawing and procedure (the authors' own creation)

One of these students has trouble expressing the result in hours and minutes. For him, 1.62 hours is 1 hour and 62 minutes. Many students have the same problem when trying to interpret the decimals in minutes. Four other students put 1:37 in their answer, which is 1 hour and 37 minutes. These students are clearer about how they go from the sum of the speeds to the result of 60/37 (see **Figure 4**).

Students who arrive at the Fibonacci solution, 1 hour and 23/37 or 1.62 hours, simply solve the problem mathematically, without questioning the context or the possibility of the situation.

Correct Answers – Different from Fibonacci's

The solutions that differ from Fibonacci's one, but are considered correct, were 2 and 5 hours. How students got these times will be presented in what follows.

These answers are assumed to be correct because they make valid considerations in the real world. The realistic consideration they make is that these animals cannot be together. Therefore, they propose dividing the sheep equally and then giving each animal its share separately. This is a different model than the one proposed by Fibonacci, where each animal will eat different amount of the sheep. These are the realistic considerations that the students consider when solving the problem, leading them to answers different from Fibonacci's, but perhaps more acceptable in real life.

Equitable distribution: Sequential consumption

Twelve students propose dividing the sheep equally, giving each animal its share one after the other, which implies sequential consumption. Whole "eating time" is the sum of "partial eating times" each animal used to eat its share. Ten students give the solution of 5 hours. The other two students follow the same idea of dividing the sheep into three equal parts, but due to rounding, give the answer of 4.9 hours (see **Figure 5**).

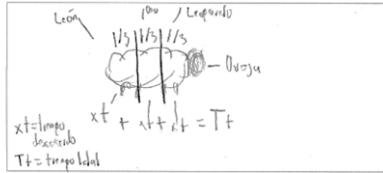
The lion takes 4/3 hours to eat its share. Next, the leopard takes 5/3 hours to eat its share. Finally, the bear takes 6/3 hours to eat its share. The sum of the times for sequential consumption is 5 hours. However, the sequential consumption is not explicitly mentioned by the students. Another student performs two procedures and arrives at the answer of 5 hours (see **Figure 6**).

Although the numerical solution is the same in both procedures, the modelling is not. In one procedure, they divide the sheep equally, as the 11 previous students do. In the other, they only average the times, which is considered an intuitive answer with incorrect modelling. When students take real-life considerations into account, they assume an equal division and sequential consumption of the sheep because they believe, according to common-sense reasoning, that a lion, a leopard, and a bear cannot be together, much less share the food. Consequently, their solution differs from the one given by Fibonacci. The solution of 5 hours, in which the students resort to an equal division, results from adding the times in which each animal eats its share, since consumption is sequential, one animal eats after the other. The same modelling (equal division and summation of consumption times) has been observed in other similar problems (Etcheverry & Slisko, 2021).

Plan: What I would do is give each animal a third of a sheep, record the times and add them up; you could also use an average, but you convert the hours into minutes to make adding and dividing easier.

Drawing:

Pon abajo un dibujo (o un esquema o una gráfica) que haría más claro tu plan.



Procedure:

4 hours = 240 minutes $\div 3 = 80$ minutes
 5 hours = 300 minutes $\div 3 = 100$ minutes
 6 hours = 360 minutes $\div 3 = 120$ minutes
 300 minutes = 5 hours

Answer: 5 hours

Verification: If you assign one-third of a sheep to each animal, logically, it should take them one-third of the total time; therefore, to find the time it takes for the three of them together, you need to add the time each one takes and convert that total into hours.

Figure 5. Student 1.4D's drawing and procedure (the authors' own creation)

Plan: I divided the time each one took to eat by 3 and added the results together ($1.3 + 1.6 + 2 = 4.9 \approx 5$); so, the three of them took approximately 5 hours. Another way to get the result is to add the hours ($4 + 5 + 6$) and divide by the 3 animals ($15 \div 3$), which also yields 5 hours.

Drawing:

Pon abajo un dibujo (o un esquema o una gráfica) que haría más claro tu plan.



Procedure:

$\frac{1}{3} \frac{4}{x} = 1.3$
 $\frac{1}{3} \frac{5}{x} = 1.6$
 $\frac{1}{3} \frac{6}{x} = 2$
 1.3
 $+ 1.6$
 2.0
 $\frac{4.9}{3} \approx 5 \text{ hrs}$

$4 + 5 + 6 = 15$
 $\frac{15}{3} = 5 \text{ hrs}$

Answer: 5 hours

Verification: If there is a single sheep among three animals ($1/3$), you must calculate the time for each animal (4, 5, and 6) to determine the figure; the individual time for each is derived from the sheep's quantity, and summing these times yields $4.9 \approx 5$ hours—or, simply by adding the hours for the three animals and dividing by three, you get 5 hours.

Figure 6. Drawing and procedure of student 5.4B (the authors' own creation)

Equitable distribution: Simultaneous consumption

Four students propose dividing the sheep into three equal parts and giving each animal its share at the same time. They explain that the time it takes them to finish the sheep is the same as the time it takes the bear, the slowest animal, to finish its share, that is, 2 hours. This proposal makes sense, since the question asks for the time, it takes them to eat a sheep together and finish it completely, as this last student correctly indicates, when the slowest animal finishes its share (see **Figure 7**).

The 2-hour solution, which also assumes an equal distribution, shows simultaneous consumption. Since each animal eats its share of the sheep at the same time, the consumption time is the time it takes the slowest animal, the bear, to eat its share. By giving these solutions, the students consider the feasibility of the situation and highlight the potential negative consequences of artificial contexts for problems used in school mathematics (Slisko, 2014).

The answer to the second research question is: The solutions that differ from Fibonacci's one but might be considered as correct are "5 hours" and "2 hours".

The first solution is based on a possible idea: Each animal eats one third of sheep sequentially and the "consummation time" is equal to the sum of "partial consummation times". The second solution uses another

Plan: Divide the sheep into three parts, feed them to the animals, and observe how long it takes for them to be eaten; the time taken by the one that takes the longest is the total time required.

Drawing:



Procedure:

$$\begin{array}{ccc} 60 \times 4 = 240 & 60 \times 5 = 300 & 60 \times 6 = 360 \\ 80 \text{ minutes} & 100 & 120 \\ 1:30 \text{ h.} & 1:40 & 2:00 \end{array}$$

Each one will take

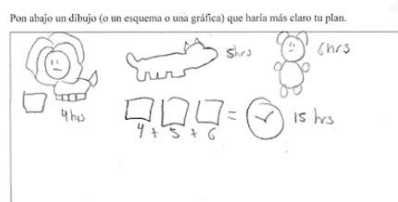
Lion	leopard	bear
1:30 h	1:40 h	2:00 h

Answer: 2 hours

Figure 7. Student 2.4D's drawing and procedure (the authors' own creation)

Plan: Well, first, calculating the times lion - 4 h, leopard - 5 h, bear - 6 h (each takes an hour longer than the last) I figure that if I add the three times together and divide by 3, it comes out to 5 hours, which is the calculated time it takes for each of them to finish eating.

Drawing:



Procedure:

$$4 + 5 + 6 = 15 \quad 15 \div 3 = 5 \text{ h}$$

Answer: 5 hours

Verification: Well, I added them all up since there are three of them and the question asks for the total for the three of them, not for each individual one; that's why I added them up and divided by three.

Figure 8. Drawing and procedure of student 3.2D (the authors' own creation)

possible idea: Each animal eats one third of sheep simultaneously and the "consummation time" is equal the "consummation time" of the slowest animal (the bear).

It is interesting to report the incorrect solutions, too.

Incorrect Solutions – Intuitive

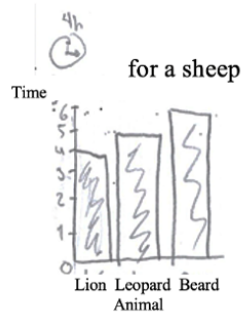
The 5-hour solution is considered a quick or intuitive response (Etcheverry & Slisko, 2021; Kahneman, 2011), and this is the answer given by 28 students. They simply calculate an average time without providing any real argument beyond simple addition and division. This could also reflect the "didactic contract" (Ávila, 2001; D'Amore, 2011). **Figure 8** shows an example.

Incorrect Solutions – Other

Regarding other solutions, we will begin with the students who use fractions but do not arrive at the correct result. Two students give the solution of 1.2 hours. The reasoning of both students is correct, but

Plan: If a lion eats a sheep in 4 hours, a leopard in 5 hours, and a bear in 6 hours, one could calculate the consumption rate for each by dividing 1 by the time in hours each animal takes and then summing those results.

Drawing:



Answer: 1.2 hours

Procedure:

$$\begin{aligned} \text{Sheep / hour} \\ \text{Lion} &= \frac{1}{4} = 0.25 \\ \text{Leopard} &= \frac{1}{5} = 0.2 \\ \text{Bear} &= \frac{1}{6} = 0.16667 \\ 0.25 + 0.2 + 0.16667 &= 0.61667 = 1.2 \text{ h} \end{aligned}$$

Verification: It would take the lion, the leopard, and the bear roughly 1.2 hours to eat a sheep together.

Figure 9. Drawing and procedure of student 3.A (the authors' own creation)

Plan: If three sheep are supposed to be eaten in that time, the average would be five; however, if only one is eaten, the average is divided by three to determine the time it takes to eat a single sheep.

Drawing:



Procedure:

$$4 \text{ h} + 5 \text{ h} + 6 \text{ h} = 15 \text{ h} \div 3 = 5 \text{ h} \div 3 = 1:40$$

Answer: 1 h 40 minutes

Verification: I divided the average because it needs to be divided to calculate the average for eating a single sheep.

Figure 10. Procedure and drawing of student 7.2D (the authors' own creation)

something happens with their execution in the last operation. This causes them to not arrive at the correct result (see [Figure 9](#)).

Five students calculate the average time it takes for three animals to eat one sheep. They know it must take less than 5 hours, since they will only eat one sheep between the three of them, so they divide this result by 3. The solution is the same as that of the students who propose dividing the sheep, 1 hour and 40 minutes, but their modelling of the problem is different, since they do not propose an equal division of the sheep. [Figure 10](#) shows the procedure of one of these students.

One of these students does not put the answer of 1.66 hours or 1 hour and 40 minutes, because, like many other students, she or he does not know how to interpret the decimals of his answer and puts the result of 2 hours, because she or he interprets 0.6 as one more hour (see [Figure 11](#)).

Sequential consumption

Thirteen students have the idea of adding the fractions $1/4$, $1/5$, and $1/6$. This is considered sequential because they view these fractions as representing time. Some arguments are as simple as converting to fractions, which doesn't explain anything about the context. Others explain why they convert to fractions, which they call the consumption rate, according to their understanding of the problem. Five of these students

Plan: I'm going to add up the three numbers which equals five and divide by three; $5 \div 3$ equals 1.6 hours, but since 1.6 involves 60 minutes, I'm setting it at 2 hours.

Drawing:

Pon abajo un dibujo (o un esquema o una gráfica) que haría más claro tu plan.



Answer: 2 hours

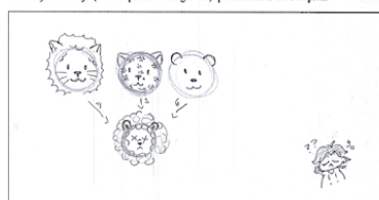
Verification: The animals will eat their share of the sheep, and they will do so quickly in 1.6; but 1.6 amounts to 60 minutes so it is 60 minutes, because that makes two hours.

Figure 11. Drawing and procedure of student 8.1B (the authors' own creation)

Plan: I think I'll set up the problem using fractions, then add them up to get the result. Or at least, that's what I hope. ☺

Drawing:

Pon abajo un dibujo (o un esquema o una gráfica) que haría más claro tu plan.



Procedure:

$$\frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{1}{4} + \frac{6}{30} + \frac{5}{30} = \frac{1}{4} + \frac{11}{30} = \frac{15}{60} + \frac{22}{60} = \frac{37}{60}$$

$$\frac{37}{60} = 0.616$$

Answer: 0.6 hours

Verification: The sums of the fractions are correct, but their representation in hours is very likely incorrect.

Figure 12. Procedure and drawing by student 4A (the authors' own creation)

give an answer of 0.6 or 0.61 hours, which is incorrect, since that's only what they eat in one hour. **Figure 12** shows the production of one of these students.

Five other students follow the same procedure. They add the fractions and arrive at the same solution, but they interpret their result differently, hence their different solutions. One student arrives at the solution of $\frac{37}{60}$ but writes 1 hour in her or his solution. Four students correctly add the fractions and arrive at the result of 0.61 or round it to 0.62, but their answers are different. One writes 1:30 hours, another interprets it as 1:10 hours, another as 1 hour and 1 minute, and the last one interprets it as 1 hour and 2 minutes. The last three students make other types of errors. One of them makes a mistake when adding the fractions, and their solution is $\frac{37}{30}$. The other two students make a mistake when converting the decimals to minutes. However, they do not show the operations they used to make this unit conversion.

These last 13 students understand the use of fractions and can even explain that they represent the consumption rate of each animal. However, they are unclear about what the solution represents. Their pictorial and/or graphical representations do not help them understand the result, nor do they influence them in any way, although they do have an idea of what they need to do. Ultimately, they need to know that this solution only represents the portion of the sheep that is eaten in one hour. The students have many difficulties with unit conversion and the interpretation of decimals.

Plan: Divide the sheep into three parts and then consider the time it takes for each animal. This yields different durations one for each animal and if they were to eat the sheep together, one would benefit more than the other, so we would calculate an average.

Drawing:

Pon abajo un dibujo (o un esquema o una gráfica) que haría más claro tu plan.



Procedure:

$$1:20 \text{ h} + 1:40 \text{ h} + 2:00 \text{ h} = 5 \div 3 = 1:40 \text{ h}$$

Answer: 1:40 hours

Figure 13. Procedure and drawing by student 7.5D (the authors' own creation)

Plan: Decompose the numbers, divide them together, try dividing them, or find the least common multiple.

Drawing:



result in minutes =
minutes in hours

Procedure:

$$\begin{array}{r} 4 \ 5 \ 6 \ 2 \\ 2 \ 5 \ 3 \ 2 \\ 1 \ 5 \ 3 \ 3 \\ 1 \ 5 \ 1 \ 5 \\ \hline 1 \ 1 \end{array} \quad \begin{array}{l} 60 \text{ min} = 1 \\ \times \\ \frac{4}{1} + \frac{5}{1} + \frac{6}{1} = \frac{120}{1} \end{array}$$

$$\begin{array}{r} 4 \ 5 \ 6 \ 4 \\ 1 \ 5 \ 6 \ 5 \\ 1 \ 1 \ 6 \ 6 \\ \hline 1 \ 1 \end{array} \quad \begin{array}{l} 120 \text{ min} = 2 \end{array}$$

Answer: 2 hours

Figure 14. Procedure and drawing of student 6.4B (the authors' own creation)

Equitable sharing: Sequential consumption

Six students propose dividing the sheep equally. Two of them record the time it takes each of them to eat their share, add them together, and calculate the average, which turns out to be 1 hour and 40 minutes. The proposal from these two students is quite interesting, since it is not an average of the hours it takes each of them to eat a sheep, which is considered a quick solution (Kahneman, 2011). Their solution is the average time it takes each of them to eat one-third of their assigned sheep (see [Figure 13](#)).

Other

Another procedure, used by 10 students, is to find the least common multiple of the three times. Six of these students correctly follow the procedure and arrive at the solution of 60, which is the least common multiple of 4, 5, and 6. Two of them interpret the result as 60 hours, and the other four as 60 minutes, that is, 1 hour. One student finds the least common multiple in two ways (see [Figure 14](#)), but one procedure is incorrect and leads to the answer of 120.

Two students add an extra two and give the answer for 2 hours. The last of these ten students proposes a plan that does not follow their mathematical procedure. Apparently, they want to find the least common multiple, but they also add an extra two and instead of multiplying the numbers they obtain, they add them and divide the result by three, giving an answer of 3.3 hours. One student obtains an answer in their plan, which is 4.9 hours, but in their mathematical operations and in the verification part, they explain that they used the greatest common divisor, but they do it incorrectly, and it is unclear how they arrive at the answer of 1.5 hours.

Four students also gave the answer of 2 hours. These are examples of the "didactic contract" since they perform operations with the data without giving a valid or justified explanation (Ávila, 2001; D'Amore, 2011).

Plan: We must subtract the time taken by the slowest eating animal from the time taken by the animal that ate slightly faster; then, we subtract that result from the time taken by the fastest-eating animal, and the final result is the time it would take for all three animals together.

Drawing:

Lion Leopard Bear



Answer: 3 hours

Verification: Subtracting the previously mentioned amount leaves a result of 3.

It can also be done in the following way:

$$\begin{array}{r} \text{hrs. Bear } 6 \\ \text{hrs. Leopard } 5 \\ \text{hrs. Lion } 4 \\ \hline 2 \text{ hrs} \end{array} \quad \begin{array}{r} \text{hrs. leopard } 5 \\ \text{hrs. Lion } 4 \\ \hline 2 \\ \hline 3 \text{ hrs} \end{array}$$

Procedure:

$$\begin{array}{r} \text{hrs. Bear } 6 \\ \text{hrs. Leopard } 5 \\ \hline 1 \end{array} \quad \begin{array}{r} \text{hrs. lion } 4 \\ \hline 1 \\ \hline 3 \text{ hrs} \end{array}$$

Figure 15. Drawing and procedure of student 7.B (the authors' own creation)

One student, as explained in her or his plan, simply multiplies the data and divides by 60 to convert to hours, without any explanation. Another student calculates the average, resulting in 5, but writes 2 in the solution. Another student proposes subtracting the times, finding the difference between 6 and 5, and between 5 and 4, and adding these two results. The last student simply subtracts one hour from each of the other two animals' times from the lion's time; that is, they subtract 2 from 4, as explained in their plan. Four students give the answer of 3 hours, with different justifications. The next student's procedure clearly reflects the learning contract, as they only manipulate the numbers given in the problem (see [Figure 15](#)).

One student only divides the time it takes the lion by 3, and their answer is 1.33. Another answer, 6 hours, is given by only one student. Several errors are observed in the procedure. Another student adds up the times it takes each animal to eat a sheep, and their answer is 15 hours. In these two answers, since they are greater than the time it takes each animal to eat a sheep, the students should be able to recognize the impossibility of their answer. Two more students arrive at the answer of 1 hour, and another at 1.9 hours. However, their arguments are not coherent, and they do not perform calculations to support their results.

CONCLUSIONS

When analyzing the solutions, we realize that students arrive at reasoned answers in alternative ways, taking reality into account. Allowing students to use their own problem-solving strategies to solve word problems in math class and to make assumptions beyond those imposed by the school context can motivate them to find connections between what they learn in school and what they learn outside of it. In this way, they can use their mathematical knowledge to model reality outside of a school math context. This connection can create meaningful and lasting learning for students.

Furthermore, the different strategies lead to different results, demonstrating that a problem does not always have a single correct answer. Answers that differ from Fibonacci's should not be considered errors, as they simply reflect each student's way of thinking and are valid. This fosters the development of critical thinking, emphasizing the problem-solving process more than the mere result.

Historical problems can be used to foster discussion in the mathematics classroom by comparing the answers given by students who draw on their real-world knowledge with those who provide answers based on established mathematical rules within the mathematical context, such as the Fibonacci sequence. In this way, historical problems can demonstrate that mathematics is not simply a set of concepts to be memorized, thus humanizing the subject. This can motivate students to continue solving problems and learning mathematics.

As a possible direction for future research, it could be interesting to explore students' approaches to this historical problem in other educational settings and with students from different countries, to examine whether similar modelling patterns and realistic considerations emerge.

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