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Research Article

Impact of a mathematical pre-course on first-year physics students

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ABSTRACT

The transition from school-level mathematics to the more abstract and formal structures in universities poses challenges for many first-year physics students. To address this gap, most physics faculties offer a mathematics pre-course. Here, we report about a pre-posttest study investigating the impact of a pre-course on N = 56 first-year physics students at the Leipzig University. The according tests were conducted in October 2022, which were correlated to the subsequent first and second-semester exam results. Thus, the research focused on measuring the knowledge gain and changes in mathematical abilities before and after the pre-course as well as the resulting medium-term effects. The results show a significant improvement in the math skills of the participants in the pre-course, especially among participants with intermediate prior knowledge. Additionally, the study reveals a correlation between the level of school mathematics instruction and learning success in the pre-course. Medium-term effects revealed that pre-course participants displayed higher pass rates and better grades, particularly in modules directly influenced by pre-course contents. This research underlines the effectiveness of the pre-course in mathematics in improving and reactivating the mathematical skills of first-year physics students.

Keywords: pre-course, mathematic prior knowledge, first-year students, higher education teaching

INTRODUCTION

The transition from school to university is a major challenge for many students, especially in degree programs with a high proportion of mathematics (Biehler et al., 2011; de Guzmán et al., 1998; Holton, 2001). The first-year students show a high degree of heterogeneity regarding their mathematical skills. A standard measure to address poor prior knowledge is university pre-courses. It is common for almost all physics faculties throughout Germany to offer a mathematics pre-course prior to the actual start of the respective study program. Buschhüter et al. (2016) were also able to show this, as in their study a large majority of the universities analyzed in Germany offer a pre-course for first-year physics students. These courses can be considered a bridge between the school and university levels aiming to reduce the heterogeneity of prior knowledge of the students. A historical perspective also shows that pre-courses are being used more and more by students. For example, in a Germany-wide study in 1978, only 27% of participants had attended a pre-course, whereas in 2013 the figure was around 70% (Buschhüter et al., 2016). In addition to the pre-courses offered by the many universities, there are also cross-university pre-courses in digital form. One example of this is the VEMINT project, which provides digital learning scenarios in preparation for university studies (Bausch et al., 2014). Internationally, pre-courses are also offered, but these are not necessarily comparable to those in Germany. Some courses aim to address the unequal prior knowledge of first-year students (Galligan, 2004), whereas other courses aim to familiarize students with the connection between mathematics and physics (Tan et al., 2017).

STATE OF RESEARCH AND DERIVATION OF RESEARCH QUESTIONS

Mathematical ability is a significant factor for success in physics-related study programs (Hazari et al., 2007; Hudson & McIntire, 1977; Hudson & Rottmann, 1981; Okey & Charles-Ogan, 2017; Popanao et al., 2023). Accordingly, it is problematic if these skills are deficient. School mathematics is usually motivated by everyday references, for instance, attention is paid to visualizing mathematical problems as clearly as possible to avoid misconceptions (Tall, 1992). However, first-year physics students often lack basic and visualization skills (Nasir et al., 2018). In contrast, university mathematics is much more oriented towards definitions and formal structures (Almeida 2000; Leviatan, 2008; Tall, 1992). This represents a change in the way of thinking and needs to be supported.

Across many STEM degree programs, lecturers agree that basic knowledge of elementary functions, predominantly linear and quadratic functions, arithmetic, and vector calculus is essential for first-year students (Deeken et al., 2020).

Research into the level of knowledge at the beginning of physics studies was carried out in Germany as early as 1978 by Krause and Reiners-Logothetidou (1981). At that time, the entrance test contained 45 items on mathematics and 47 items on physics to assess the prior knowledge of first-year students. The items in this test primarily addressed calculation skills and less complex tasks (Heinze et al., 2020). The test was carried out again in 2013 with first-year physics students. This showed that the students were able to solve some topics better compared to the test in 1978 while results declined in other parts. Overall, however, there was no evidence that the level of prior knowledge had deteriorated or improved. The topics of equations, graphs, fractions, radians and trigonometry in particular show potential for improvement (Buschhüter et al., 2016).

Due to various changes in the education system, the test from 1978 is not necessarily as meaningful today as it was at the time. For this reason, this and other tests were examined for their complexity in a meta-study and categorized according to the Knowledge for Undergraduate Mathematics (KUM) model (Rach et al., 2021). In addition, an expert survey was conducted in which experimental physics professors sorted the items of the test and other items from a task pool according to necessity for first-year students. It was found that, according to the experts, it is mainly basic knowledge that is required and only a few high-level tasks. However, the authors of this study also emphasized that this was conducted from the perspective of experimental physics and may not necessarily apply to mathematical training with less application relevance (Gahrmann et al., 2023).

In particular, students whose prerequisites are not considered optimal show significantly better performances in the first year of study due to a mathematical pre-course (Derr et al., 2018). Further, pre-courses in mathematics contribute to a better pass rate in the first semester (Johnson & O'Keeffe, 2016). In addition to the effects of pre-courses, prior knowledge tests are a good predictor of academic success in the first year (Greefrath & Hoever, 2016). Another good predictor is final school grades (Trapmann et al., 2007). The question of which groups of pupils benefit more or less from the pre-course has not yet been answered in these studies.

As part of our study, we asked ourselves the following research questions:

1. Does the pre-course we designed promote students' mathematical skills, especially the computational skills which are needed in (experimental) physics, in the short and medium term?
2. Are there individual groups of students who particularly benefit from our pre-course?

STUDY CONTEXT AND DESCRIPTION OF THE PRE-COURSE

The pre-course is primarily aimed at first-year students on the B.Sc. Physics and B.Sc. Meteorology programs at the Leipzig University, which is a larger university in Germany. Around 80 physics students and

40 meteorology students began their studies in the degree programs, with around half of the students from each degree program taking part in the pre-course. Participation in the pre-course is free of charge and voluntary for students.

The pre-course at our faculty has been taking place for over 30 years. In recent years, we have noticed lower numbers of participants, especially during the course period. For this reason, we have jointly considered how we can revise the course. To this end, we also spoke to former participants of the course and made some changes to the general procedure as well as the mathematical topics. The students´ main criticism of the pre-course was the abstract level of the mathematical content. It was criticized that the content had little to do with school content. The lack of differentiation was also noted, as the students' prior knowledge is very heterogeneous.

Many of the pre-courses take place over a period of two or more weeks. However, our pre-course is limited to one week. During the week, there are three 90-minute lectures each day. These are divided into lectures and tutorials. The content from school is repeated in a total of eight lectures. At the same time, an outlook is offered at a few points, for example on Taylor series as an application of differential calculus. The focus here is on repeating the mathematical content and calculation techniques, which we suppose to be known from the school level. The translation of the content from school into the more formal language of mathematics is also demonstrated with the students using illustrative examples. The content of the pre-course is based on the recommendation of the physics departments (Konferenz der Fachbereiche Physik, 2011), which is why the focus of the pre-course is on functions, differential and integral calculus and geometry, including vector calculus. Topics such as probability theory or complex numbers were not covered in the pre-course, as these are either less relevant for the first semesters (probability theory) or are introduced during the courses in the first semester (complex numbers) and are therefore not a repetition and consolidation of the school material.

In the exercises, the focus is on more activity for first-year students. The students are particularly involved here, as they repeat the calculation techniques from the lecture and apply them in various tasks. Students are asked to solve the problems themselves/in pairs using the knowledge from the lecture before a tutor discusses the solution with them. The Think-Pair-Share method was often used, especially to solve difficult tasks. The tasks in the exercises were at different levels, according to the KUM model (Rach et al., 2021). This is intended to enable differentiation within the heterogeneous group of first-year students. In order to create the best possible learning environment, the exercise groups are separated into much smaller groups of around 20 people than in the lecture based on their degree programs. This is intended to promote dialogue between students.

After the pre-course, an orientation week takes place until the start of the degree program. During this week, all students take part in leisure activities to establish social contacts and also receive brief introductions to the organization of their studies. No subject-specific content is taught and participation in these events is voluntary, but is usually used by almost all first-year students.

METHODS

Study Design

The prior mathematical knowledge of first-year students is very heterogeneous. The diversity of the content covered in mathematics lessons is reflected in a study by the Conference of Physics Departments in Germany (Konferenz der Fachbereiche Physik, 2011). For this reason, it is very important to determine the level of prior knowledge before an intervention is analyzed, in our case the pre-course. For this reason, we opted for a pre-posttest study.

We are aware that participants in the pre-course can also improve their mathematics skills through factors other than the knowledge transfer in the pre-course. In order to be able to draw conclusions about the pre-course at a later stage, we were looking at a control group that does not take part in the pre-course but otherwise received the same offers and learning opportunities after completing the pre-course. This means that the two groups are comparable after the course intervention. However, we would like to note that this does not imply that the prior mathematical knowledge of the target group and control group is exactly the

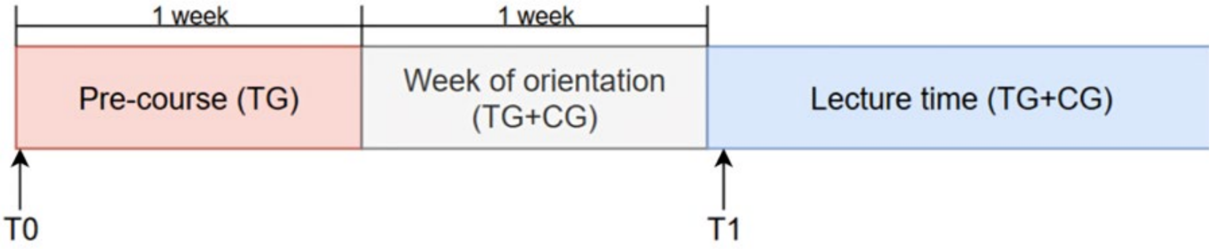

Figure 1. Timing of the pre-posttest study in the winter semester 2022/2023. The target group (TG) took the test at time T0 at the beginning of the pre-course. The target group and the control group (CG) then took the test at the beginning of the lecture time at time T1. Both groups were able to take part in the week of orientation. (Source: Authors' own elaboration)

same. However, this procedure ensures comparability since only if both groups have comparable prior knowledge, conclusions can be drawn.

Sample and Data Collection

We tested and evaluated N = 56 students in October 2022 at the Faculty of Physics and Earth System Sciences at the Leipzig University. The pre-test took place at the beginning of the pre-course at time T0. After the course, the students had one week for orientation before the lecture period began. The post-test was conducted at time T1 during the first week of lectures (see **Figure 1**). The questions were not analyzed after the pre-test in order to avoid a significant bias in the post-test. According to Solomon (1949), a 4-group design would be optimal to minimize this bias. However, this was not possible here as the resulting cohorts were too small.

Both tests were carried out in the lecture and on paper. This ensured that no aids were used and that the students worked individually and wrote the test under the same conditions.

As the test was written anonymously, it is not possible to link the results to the exam results of a specific person. We recorded the participants by name during the pre-course and were thus able to clearly assign which person took part in the pre-course and which did not. This makes it possible to categorize the target group and the control group without violating the anonymity of the test. At the same time, we were able to see that the number of participants in the course remained very stable throughout the week.

To record the pass rates and grades, the individual result lists were sorted and analyzed according to the two groups. For this purpose, we worked together with the relevant lecturers of the modules, also in order to gain insight into the corresponding tasks of the examinations and also to obtain the examination results. By recording the names of the students present at the test times T0 and T1, we were then able to ensure that we only looked at the test results of students who had previously taken part in the pre- and post-test.

Instruments

We conducted our knowledge assessment using a test sheet. The test initially included questions on demographic data, including age, gender and level of mathematics education. In addition, we assessed their own mathematics skills on a scale of 0% to 100%. Finally, the voluntary nature of the test and the use of the results for research purposes were emphasized. The test was completed anonymously so that no conclusions can be drawn about the individual. An anonymization code was used to enable the pre-test and post-test to be matched later.

For this study, we developed a mathematics test consisting of 18 items in an open-response format. No solutions, i.e., in multiple choice format, were given to minimize the probability of guessing. The students were informed that their participation was voluntary and anonymous and that the test results would be used for research purposes. The mathematics skills assessment test was based on the large-scale physics entrance test conducted in Germany in 1978 (Krause & Reiners-Logothetidou, 1981). We only adopted some of the topics. Since not all topics are taught in schools anymore and Gahrmann et al. (2023) showed that not all topics are relevant from an expert's point of view, we excluded topics logic and complex numbers. At the same time, we focused on a broad range of topics across the entire school period. This includes basic tasks for solving equations, but also more challenging topics such as integral calculus. The test was also designed not only for geometry and analysis, but also to include linear algebra. In line with the KUM model and expert opinions (Gahrmann et al., 2023), different levels were incorporated, with the majority of the tasks not being highly complex. As a result, there are individual tasks in a high requirement range. For example, the task on trigonometry (task 4), which contains a generalization, and the task on integral calculus (task 11), which requires a linear substitution, which is not taught in the majority of schools and requires a deep understanding of integral and differential calculus.

Prior to the study, several teachers optimized the tests to ensure clearly formulated tasks.

The test contains only 18 items, which significantly lower number than the 45 items in the 1978 entrance test or other tests. This also results in a shorter test completion time. We have chosen this rather small number of items in order to achieve a high response rate for the test and also a high quality of answers since it has been shown that the length of a survey is related to the quality of the answers (Herzog & Bachman, 1981). In addition, Krosnick (1991) was able to show that the quality and response rate of surveys depend on three factors: task difficulty, participant ability and motivation. Since we cannot change the abilities of the participants and the task difficulty cannot be selected at the lowest level, the motivational component must be addressed. In particular, a high response rate is necessary for a usable result, as we are looking at a rather small cohort.

Evaluation Methods

A coding manual was used to ensure comparable evaluations of the student solutions. Two points were given for a correct answer, one point for a partially correct solution, and zero points for an incorrect solution. To evaluate a partially correct solution, we mainly looked at whether the skill examined in the item was present. For instance, one point was given when calculating the cross-product, which resulted in exactly the negative vector. The students had no time limit for solving the given problems, but the estimated average time was 30 minutes. Consequently, no distinction was made between incorrect answers and items that were not answered. Two independent persons initially rated one-tenth of the tests to determine the interrater reliability. A high agreement of the ratings was shown by Cohen's kappa value of $\kappa = 0.99$ (Cohen, 1960). Due to this consistency, the coding manual was considered suitable (**Figure A1** in **Appendix A**).

For this study, only students participating in both tests were considered for the evaluation ($N = 56$) corresponding to 81% of the pre-course participants. The students' pre- and post-tests could be related via an individual anonymization code. We would like to note that during the pre-course, we could only analyze students' participation, but not the activity and extent of their participation. Students who did not attend the mathematical pre-course but participated in the post-test at the beginning of the semester were used as a control group. The students who took part in the pre-course but did not take a post-test showed comparable results in the pre-test to the test participants who were tested twice, so that we can rule out a bias in the raw data set.

We evaluated exam results from the first two semesters to investigate short- and medium-term effects of the pre-course. In the first semester, students have a module on mathematical methods in physics. This module covers topics such as differential equations, complex numbers, and matrices. These topics are usually not part of the curriculum in Germany, the United Kingdom, or the USA and are also not part of the mathematical pre-course evaluated here (Department for Education, 2021; Konferenz der Fachbereiche Physik, 2011). In addition, they take modules in experimental physics in each of their first two semesters, in which mathematical skills competencies are required covered in the pre-course.

In order to verify the validity and internal consistency of our test instruments, we determined the discriminatory power of all items. We further determined the Cronbach's alpha value to be 0.87, which means that we can classify the instrument as well suited on the basis of this value (Nunnally & Bernstein, 1994). The calculation of the discriminatory power of all items shows that almost all items have a positive value of around 0.5. Only the item for calculating a vector between two points (0.29) and for sketching the quadratic function (0.30) are just below or equal the value of 0.3.

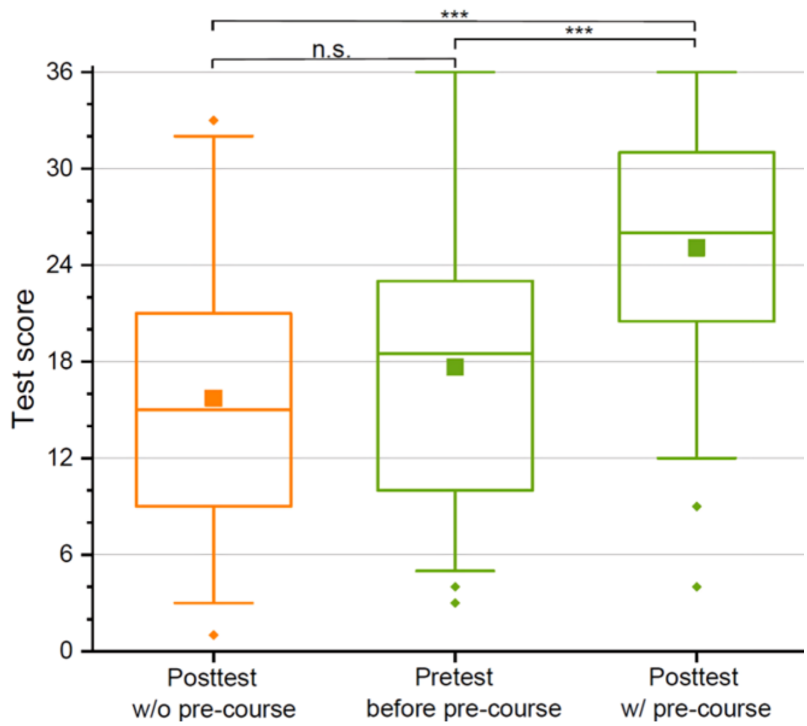

Figure 2. The results of the pre- and post-tests are shown. The same cohort completed the pre-test before the pre-course and the post-test with the pre-course (N = 56; green boxes), the second cohort (N = 57, orange box) consists of students who completed the post-test without the pre-course. There is no significant difference in prior mathematical knowledge between the pre-course participants and non-participants. After the pre-course, the students' level of knowledge improved significantly. The significance test was conducted with a Kolmogorov Smirnov test with two samples (*p < 0.05, **p < 0.01, ***p < 0.001, n.s.-not significant). The middle bars mark the respective median, the square the mean, the whiskers the 5%-95% intervals, and the diamond symbols the outliers. (Source: Authors' own elaboration)

RESULTS

Based on the results of the pre-posttest study, we evaluated changes, i.e., learning progress, and investigated medium-term effects via exam performances. The distribution of correctly solved items provides a non-normally distributed statistic, as shown by a Kolmogorov-Smirnov test with Lilliefors correction below 0.1%. The overall results are displayed in **Figure 2**. The respective first testing of participants of the pre-course (pre-test before pre-course) and non-participants (post-test w/o pre-course) revealed no significant differences in prior mathematical skills. There are also no clear differences in the demographic composition between the two groups (see **Table B1** in **Appendix B**). For the group participating in the pre-course as well as in both tests, a significant difference between the results of the pre- and post-test was measured. **Appendix C** lists all tasks set in the pre-course test.

Since students with a low score on the pre-test can improve significantly more than students with an initial high score, the individual change cannot be measured by the difference in scores between the surveys alone. For this reason, we used the Hake Index (Hake, 1998) as a measure of learning growth and for a relative score metric. For the overall pre-course, the Hake Index is $g = 0.396$, which corresponds to an average learning gain. To gain deeper insights, we further analyzed and compared the Hake Index for each student, i.e., we calculated a personalized g-value (Hake, 1998) as follows:

$$g = \frac{p_{\mathrm{post}} - p_{\mathrm{pre}}}{1 - p_{\mathrm{pre}}}. \quad (1)$$

Here, p_{post} indicates the proportion of points per person in the post-test to the total score p_{pre} for pre-test, respectively. Based on this calculation, a distribution of g-values arose (see **Figure 3**).

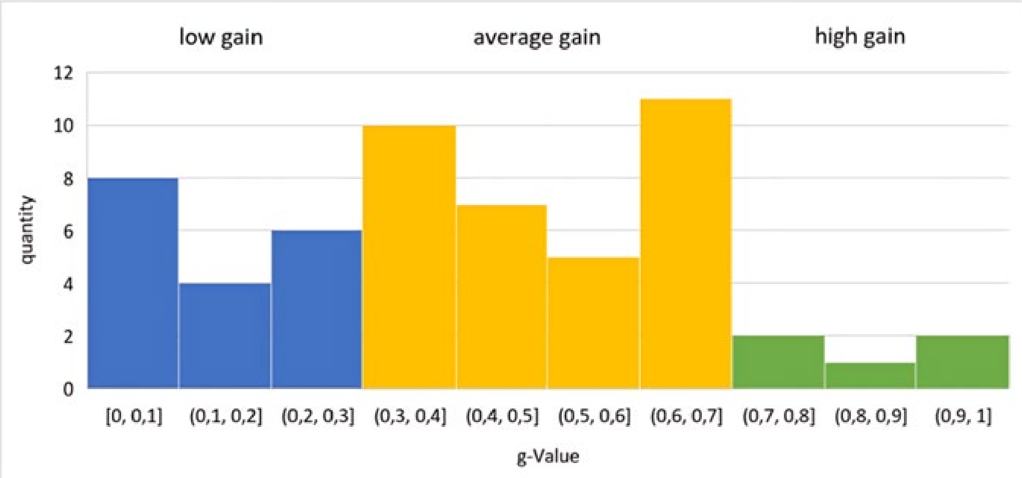

Figure 3. Representation of the frequency per calculated g-value using Eq. (1) for the participants of the mathematical pre-course (Source: Authors' own elaboration)

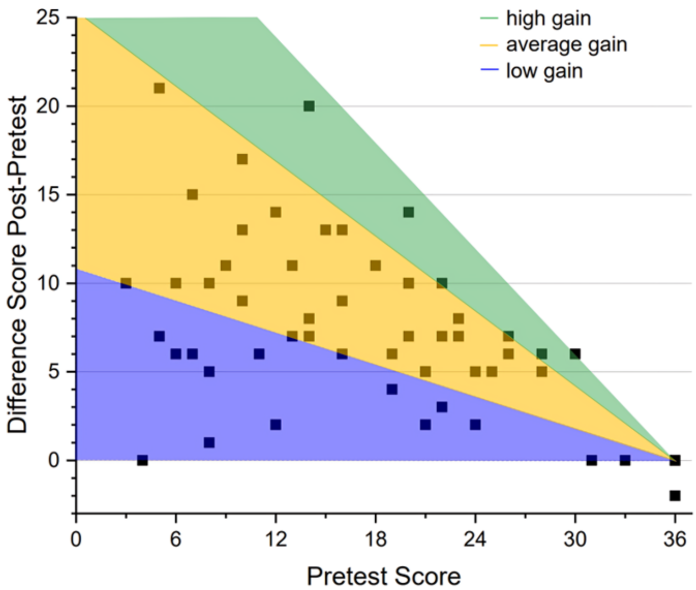

Figure 4. Illustration of the change in score between post- and pre-test. The colored areas indicate a high (green), average (yellow), and low (blue) learning gain. The learning gain was calculated using Eq. (1). (Source: Authors' own elaboration)

We clustered the distribution into three ranges with differing learning gains according to Hake (1998): $0 \leq g < 0.3$: low gain in learning, $0.3 \leq g < 0.7$: medium gain in learning, and $0.7 \leq g \leq 1$: high gain in learning. In the pre-course studied here, 60% of the participants received a medium increase and 9% a large increase in their skills. However, the distribution does not indicate which students show a stronger or weaker increase in knowledge, which is subsequently visualized in **Figure 4**.

To relate **Figure 4** to **Figure 3**, we have colored the same areas that indicate the three ranges of performance increase. The tasks in the test vary in complexity. Overall, we only see three tasks with a significantly higher level of complexity than the others. These are determining the general solution to a trigonometric problem (4.), calculating an antiderivative using linear substitution (11.) and sketching a chained function (13.). For this reason, we believe that a person with a score above 30 must have a very high level of prior mathematical knowledge. In addition, 6 items are very basic for solving equations and sketching standardized graphs apart from the logarithm function. I.e., a person with a score of about 12 has some prior mathematical knowledge, but no real in-depth knowledge. For example, a person with a score of 24 has good basic prior knowledge and some knowledge at A-level, but the person still shows clear gaps in tasks of higher complexity. In our opinion, a test score of around 24 would be a good starting point for studying.

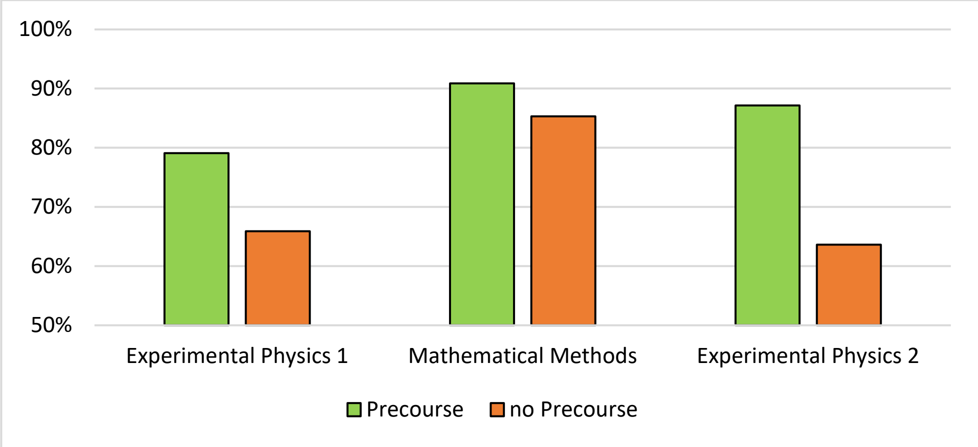

Figure 5. Percentage of pass rates in the first two semesters following the pre-course in three selected modules that build upon the content covered in the pre-course. It reveals a higher pass rate for preparatory course participants across all three modules. (Source: Authors' own elaboration)

No increase was observed in the already high-performing pupils (over 30 points in the pre-test). The group with an average pre-test level (13 to 30 points in the pre-test) showed a large gain in performance after attending the pre-course. This con be deduced from the fact that 67% of the people in the group had a medium gain and a quarter of the group had a large learning gain. The increase in performance in the group with weaker pre-test results (12 or fewer points) was very heterogeneous. This can be seen from the fact that almost the same number of people had a small or medium learning gain.

The results presented so far show solely short-term effects of a pre-course. To evaluate medium-term influences, we correlated the exam results after the first and second semesters with the participation of the pre-course. One criterion was the pass rate, i.e., how many students successfully completed the exam. In **Figure 5**, we list the pass rate in the respective modules stratified by participation in the pre-course. In the physics laboratory course all students passed, and we thus refrained from listing the module in **Figure 5**.

The pass rate for a module merely indicates the proportion of students who could not meet the minimum requirements. We further examined the grade distribution to gain insight into performance beyond the minimum requirements. In the exams, grades were given on a scale from 1.0 (very good) to 4.0 (sufficient) and 5.0 (failed), similar to the A-F grading scheme. We only considered the passed grades to eliminate the effect on passing the exam that has already been shown (see **Figure 6**). This shows whether the pre-course contributes to better grades. In order to compare the grades, we have included the physics laboratory module as a further parameter for comparison. In this module, students have to carry out experiments in the field of mechanics and then analyze them. Mathematical skills, especially those covered in the pre-course, play a minor role. This makes it possible to compare between modules whether possible effects occur in all modules or only in certain modules.

The demographic data were evaluated to identify possible groups that benefited particularly from the pre-course. Including the g-value as a measure of improvement, individual student groups could be checked for differences in learning success (see **Table 1**). **Table 1** shows that the level of mathematics instruction has a significant influence on learning gains in the pre-course. With a pooled standard deviation of 0.239, this results in an effect size according to Cohen's $d = 0.67$ implying a medium effect (Cohen, 1988).

Following the cohort analyses, we evaluated individual subject areas and improvements through the pre-course. To compare the changes, we put the individual improvements per student in relation. For this purpose, we inserted the individual percentage scores from the pre- and post-tests for each participant into Eq. (1). This shows that knowledge of logarithms ($g = 0.27$), trigonometry ($g = 0.22$), and extrema ($g = 0.19$) has improved less over the course of the pre-course. In contrast, skills in solving quadratic equations ($g = 0.53$), the scalar ($g = 0.84$) and cross product ($g = 0.54$) and integration ($g = 0.62$) have improved significantly.

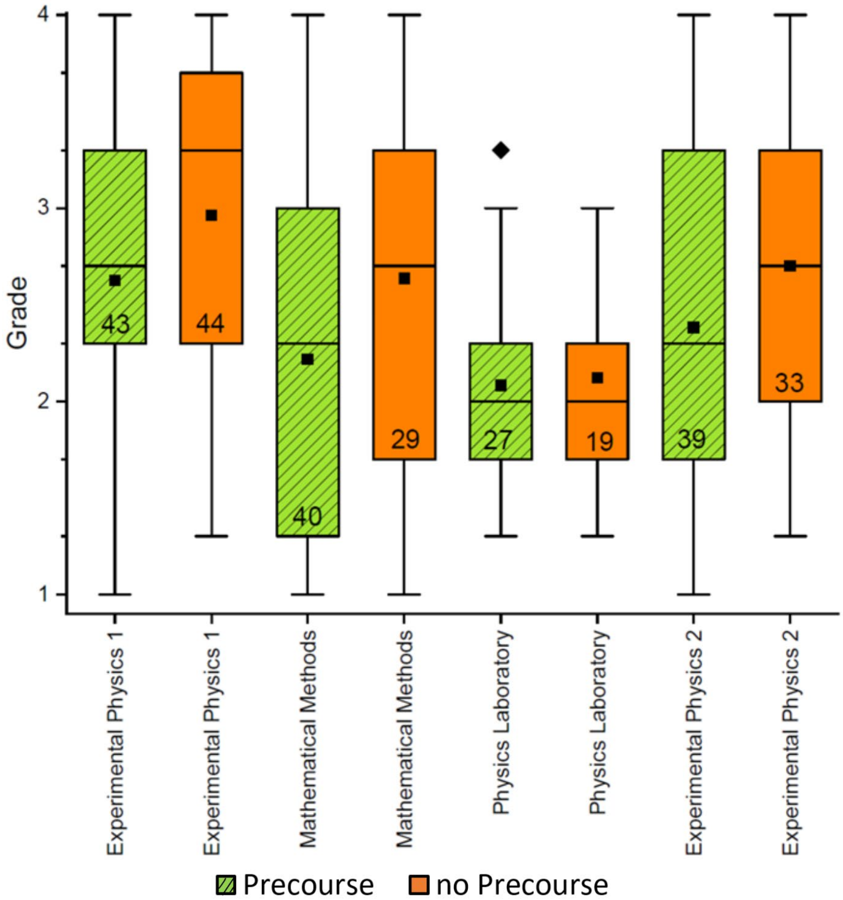

Figure 6. Representation of the grade distribution of all passed exams. The green, striped boxes represent the group of pre-course participants. The number of students for each box is listed at the bottom. The square marks the mean, the middle bars the respective median, the whiskers the 5%-95% intervals, and the diamond symbols the outliers. The pre-course participants performed better in all exams except the physics laboratory. (Source: Authors' own elaboration)

Table 1. List of individual characteristics and corresponding cohorts as well as significance test results of the characteristics. In Germany, students in the last two school years choose the level of math education between basic and advanced courses. Only the level of mathematics instruction at the school level significantly influences learning success in the pre-course. The significance test was conducted with a Kolmogorov-Smirnov test with two samples. The mean value and the standard deviation of the *g* values were specified for the individual characteristics.

Characteristic			p
Gender	Female (N = 12) 0.400 ± 0.218	Male (N = 44) 0.407 ± 0.261	Not significant 0.52352
Year of school graduation (Abitur/A-Level)	2022 (N = 29) 0.443 ± 0.291	Before 2022 (N = 27) 0.380 ± 0.186	Not significant 0.28251
Level of school mathematics education	Basic level (N = 13) 0.294 ± 0.159	Higher level (N = 41) 0.454 ± 0.259	Significant (**) 0.00391

In addition to the cognitive aspects of mathematics skills, the test instrument also asked students for their own assessment of their math skills in one item. To this end, students were asked to assess their mathematical knowledge by rating it on a scale from no knowledge (0%) to very good knowledge (100%). The correlation between the proportion of points per participant in the maximum score and the self-assessment was subsequently analyzed.

The comparison of self-assessment and students' skills are visualized in **Figure 7**. In **Figure 7**, we have shown the measured performance in the pre- and post-test in relation to the self-assessment of mathematical skills to illustrate the changes due to the pre-course. In the optimal scenario, self-assessment aligns with

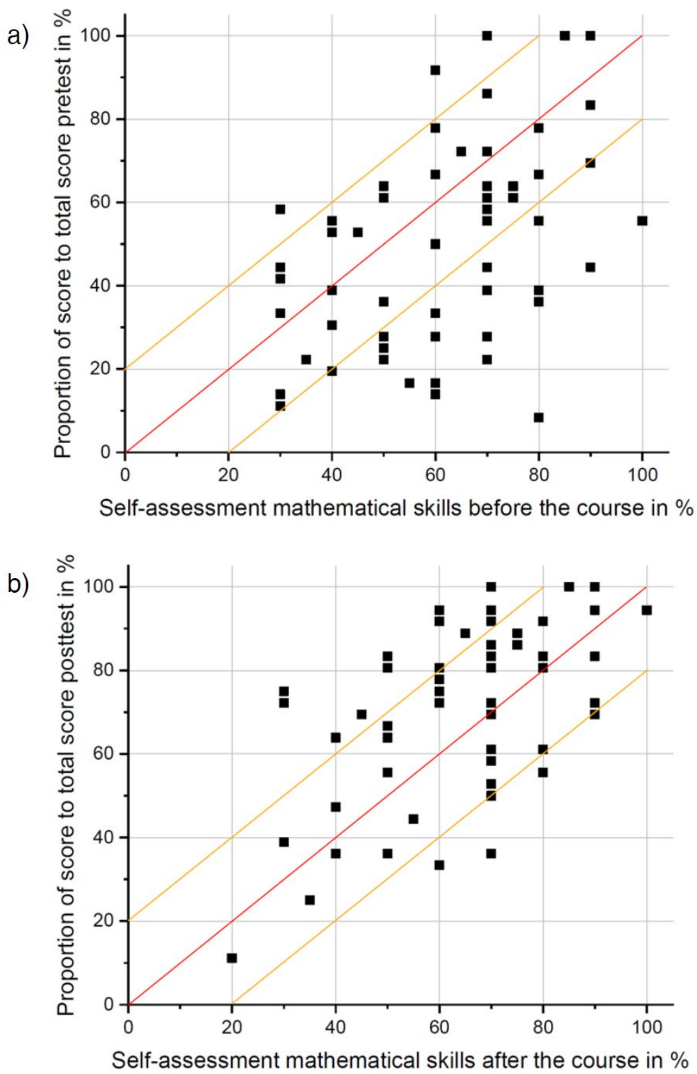

Figure 7. Comparison of the students' self-assessment a) before and b) after the pre-course in relation to their percentage scores in the respective tests. Before the course, many students overestimated their skills. After the course, the students have a much better self-assessment, but a few underestimated their skills. (Source: Authors' own elaboration)

mathematical abilities, denoted by the red line. Acknowledging the challenges of a precise assessment and seeking to identify trends rather than exact values, we have included a range of ±20% deviation from this ideal case in both directions, which is represented by the orange lines as a visual guideline for reasonable correct self-assessment. Before the pre-course, 32% of the students were below this reasonable self-assessment, i.e., they overestimated their A-level mathematics abilities. The situation changes when the self-assessment is correlated with the test results of the post-test after the pre-course. The proportion of students below the 20% deviation drops to 5%. The proportion in the interval of $\pm 20\%$ around the mean increases from 62.5% to 75%. At the same time, however, the proportion of underestimation increased slightly.

DISCUSSION, LIMITATIONS, AND OUTLOOK

The pre-post study shows a significant increase in knowledge through the pre-course. Before the course, the mathematical prior knowledge of the pre-course participants was indistinguishable compared to the non-participating cohort. In order to identify groups that specifically benefited from the pre-course, we linked the increase in knowledge to the Hake Index as a numerical value (Hake, 1998). In the original definition, according to Hake, the value is a measure of learning growth. This cannot reasonably be assumed for the presented study since parts of the topics taught in the pre-course should have also been taught in school education. It is therefore not just a matter of a pure learning gain, but also a reactivation of knowledge. The determined g-value is therefore a measure of the learning gain that can be attributed to the activation of prior knowledge through the pre-course. However, learning success is the desired metric that is directly linked to passing exams and grades.

We would like to note that one student showed a deterioration in the post-test. As the g-value is only meaningful for improvements, this individual person was not included in the distribution. As this is only one person, we consider the impact on the result to be marginal. As can be seen in **Figure 3**, around 31% of the first-year students showed no or very little improvement in the post-test. In our view, there could be several possible explanations for this. Individuals may not have attended the entire pre-course and therefore not have really benefited from it. It is also possible that some students did not have the motivation to complete the test again in the post-test and therefore made less effort. We have tried to minimize this component through the design of the test instrument, but it cannot be ruled out. With regard to **Figure 4**, there is another reason for first-year students with no improvement in the second test. Several students with very good prior knowledge showed no measurable, significant improvement. This could be due to the fact that they offer little potential for improvement with their good prior knowledge and at the same time the pre-course did not differentiate enough to further promote students with strong prior knowledge.

The pre-course did not offer any demonstrable learning gain to students with an already high level of prior knowledge and no knowledge reactivation was needed. It should be noted that the pre-course is not explicitly designed for students with excellent prior knowledge. Students with very weak prior knowledge could be supported to some extent. However, not all of them showed major improvements. Those who attended the pre-course with an intermediate level of prior knowledge have shown a substantial knowledge increase. Almost all students within this group achieved at least a medium learning gain. Students with poor prior knowledge were only able to experience a moderate improvement in some cases. The increase in the level of knowledge through the pre-course was not linked to gender nor the year of graduation. The only demographic factor that showed a significant influence is the level of mathematics instruction during the last two school years. One possible explanation could be the more rapid acquisition of complex content due to the higher level of instruction. Thus, the higher level of instruction cultivated the ability for abstraction better. This leads to a better linking of the mathematical content (Nurjannah & Kusnandi, 2021). Similarly, more content in the school courses with a higher teaching level could be the basis for a higher degree of reactivation.

However, when looking at the learning gain in the individual items, it has also been shown that some topics have a higher gain than others. The different development cannot be explained in more detail on the basis of the test data. One possible explanation would be that the scalar and cross product are significantly faster to reactivate in the reproduction of knowledge than the mastery of the laws of logarithms or the generalization of zeros for trigonometric functions. For some topics, it may also be that individual first-year students have not had these at school and have therefore learned about them for the first time in the pre-course. Due to the pandemic measures in the previous years in particular, the curricula in schools were not necessarily always fulfilled (Engzell et al., 2021). As a result, not all topics were solely reactivated, but were also completely new to the students. This could lead to a smaller learning gain if the topic is difficult, or to a large learning gain if the topic is relatively easy.

One limitation of our study design is that the target group has completed the test a second time at time T1 and is therefore already familiar with the structure and question types. We tried to minimize this factor by collecting the test after everyone had completed it and by not discussing the tasks with the students after the first test. In addition, we did not announce that there would be another test and we were not asked about this by the first-year students.

The comparison to other studies on mathematics pre-courses is not trivial since the contents of the pre-courses as well as employed tests vary significantly. One way of classifying the results is the Hake Index (Hake, 1998) as a measure of learning growth through the pre-course. Within our study, this resulted in a Hake Index of $g = 0.40$. This value is significantly higher than for courses studied previously yielding a $g \approx 0.2$ (Coletta et al., 2007). The high value of our pre-course is favored by the fact that it included repetition of school contents and not exclusively new knowledge. The pre-course rather reactivates part of the knowledge. The current study made no distinction between reactivation and new knowledge acquisition. As shown above, this clear change in mathematics knowledge becomes mainly apparent for students with an intermediate level of prior knowledge, who benefit particularly from the pre-course.

To investigate the medium-term effects of the pre-course, we evaluated the examination results in the first and second semesters. We would like to note that we are aware that there are other ways of analyzing the medium-term effects in addition to looking at exam results. There is a risk that students may develop a certain amount of fatigue when taking tests during the semester and that the quality of their answers may decrease as a result (Gregory, 2013). Thus, we have chosen to analyze the exam results. The exams do not necessarily reflect conceptual knowledge, but the majority of the calculation skills we looked at were required both in the exam on mathematical methods and in experimental physics. The two groups of pre-course and non-participants did not significantly differ in their prior mathematical knowledge (see **Figure 2**). Thus, both groups initially differ only in their participation in the pre-course. Nevertheless, the data revealed a higher success rate and better examination grades of the pre-course participants in the first and second semesters. At first glance, it could be reasoned that students who participated in the pre-course were more committed than the non-participants. However, when looking at the grades in the physics laboratory course, we noticed that the two groups were indistinguishable (see **Figure 6**), minimizing the possibility that study commitment is the sole reason. In contrast to the other modules considered, the physics laboratory course is not cantered on mathematical skills. Thus, the observed effect cannot be readily attributed to more committed students since a difference in grades should also be recognizable in this course.

In addition to assessing the students' mathematics skills, we also asked how the students rated themselves, in particular whether there had been a change after the pre-course. The recording of self-assessments before and after the pre-course shows a shift. Before the pre-course, students tended to overestimate their mathematical knowledge. After the pre-course, the students assessed their abilities more realistically. One possible explanation for this could be that self-assessment improved as a result of the increase in subject-specific knowledge (Kruger & Dunning, 1999). As the evaluation of self-assessment was recorded on a single item basis, it is not feasible to make a reliable statement about the change. This cannot be determined by the one item. The change shown requires a more targeted investigation with a test instrument specifically designed for this purpose, which we are unable to achieve with our approach. Thus, the aspect shown here offers the prospect of a need for further research in the future. Furthermore, the results shown in this publication should be further analyzed by continuously recording prior knowledge in order to be able to confirm the corresponding results. This will further improve the validity of the study and the instrument.

Based on the results, we can conclude that the pre-course we have designed promotes mathematical calculation skills but requires further revision. One possibility would be further differentiation measures. One example of this would be additional digital tasks that include both easier and more difficult tasks in order to provide a more targeted practice platform. On the 'Moodle' learning platform, it is possible to create so-called STACK tasks. These are randomized tasks with automatic feedback (Sangwin, 2013). This would allow learners to practice in an even more differentiated way and participate individually and digitally in the pre-course.

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Data availability: Data generated or analyzed during this study are available from the authors on request.

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APPENDIX A

3. Determine $x \in \mathbb{R}$ so that the equation $3 \cdot \ln(2) + \ln(3) = \ln(x)$ is true.
Correct solution: Apply the laws of logarithms correctly and obtain the correct result: $3 \cdot ln(2) + ln(3) = ln(2^3) + ln(3) = ln(8 \cdot 3) = ln(24) = ln(x) \longrightarrow x = 24$
Partially correct solution: Application of a logarithmic law correct, second law incorrect
Incorrect solution: incorrect application of the laws of logarithms
4. Give all $x \in \mathbb{R}$ for which $0 = 2 \cdot \sin\left(\frac{x}{4}\right)$ is true.
Correct solution: Specify all possible values for x: $x = 4 \cdot \pi \cdot k \;\; k \in \mathbb{Z}$
Partially correct solution: no specification of an incorrect value and specification of at least one possible value for x, for example: $x = 0$
Incorrect solution: No possible value specified for x

Figure A1. Excerpt from the coding manual (Source: Authors' own elaboration)

APPENDIX B

Table B1. Comparison of the cohort that took part in the pre-course (TG) and the cohort that did not (CG). The mathematics grade in the A-level in Germany is awarded from 15 (very good, equivalent to an A) to 0 (failed, equivalent to an F). There is no discernible difference between the cohorts in terms of math grade, graduation year and level of math instruction. There is only a small difference in the proportion of female students.

	Proportion of female students	Proportion of higher mathematics level in the A-levels	Mathematics grade in the A-levels	Year of the A-levels
TG	20%	76%	12.0	2020.91
CG	33%	78%	12.2	2020.84

APPENDIX C

List of all tasks set in the pre-course test

1. Determine the maximum number range $x \in \mathbb{R}$ for which the inequality $\frac{3}{4} \cdot x + 2 > 6 \cdot x + \frac{1}{4}$ is fulfilled.

2. Calculate the zeros of the function $g(x) = 2x^2 + 3x + 1$.

3. Determine $x \in \mathbb{R}$ so that the equation $3 \cdot \ln(2) + \ln(3) = \ln(x)$ is true.

4. Give all $x \in \mathbb{R}$ for which $0 = 2 \cdot \sin\left(\frac{x}{4}\right)$ is true.

5. $\frac{\pi}{2}$ correspond to how many degrees of a plane angle?

6. Calculate the solution to the system of equations

$$7 \cdot x + 3 \cdot y = 5 \quad 2 \cdot x - 3 \cdot y = 13$$

7. Calculate the vector $\vec{AB}$ between the points $A(-1|2|4)$ and $B(1|0|-1)$.

8. The vectors $\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$ are given. Calculate the scalar product $\left(\vec{a} \cdot \vec{b}\right)$ and the vector product $\left(\vec{a} \times \vec{b}\right)$.

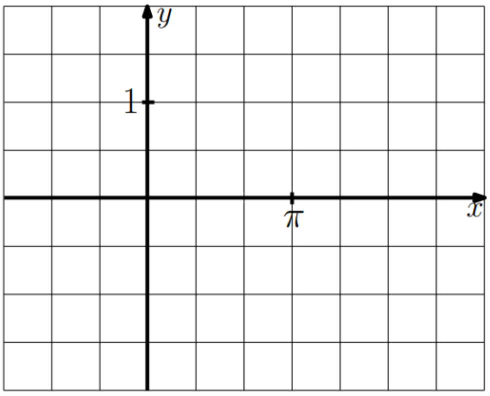
9. Calculate the first derivative of the function $f(x) = e^{2x} \cdot \cos(x)$.

10. Find the type and point of the extreme of the function $f(x) = a \cdot x^2 - 2x$, where $a > 0$ is a constant.

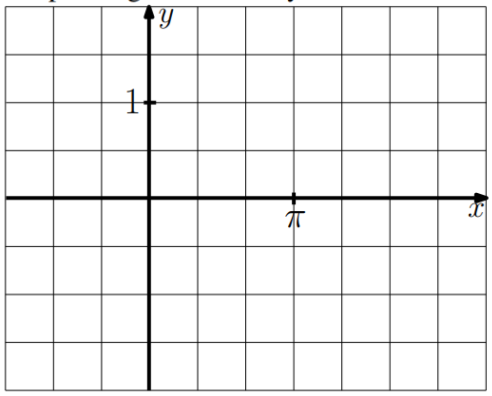
11. Calculate a primitive function/indefinite integral of $g(x) = \sqrt{2x - 1}$.

12. Calculate the solution of the integral $\int_0^2 \left(3 \cdot x^3 - 4\right) dx$.

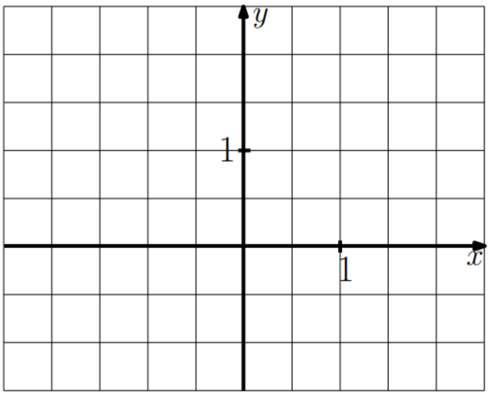
13. Sketch the functions indicated below the corresponding coordinate systems.



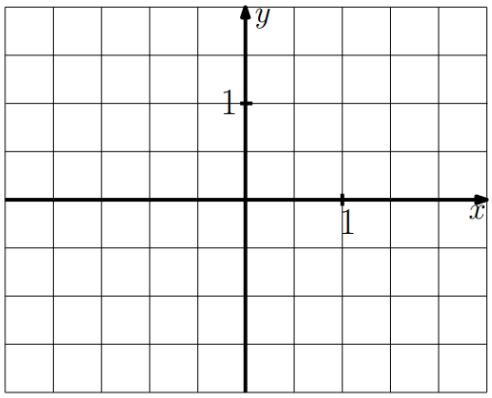
$f(x) = \cos(x)$



$f(x) = \sin^2(x) = (\sin(x))^2$



$f(x) = x^2 - 1$



$f(x) = e^x$ und $g(x) = \ln(x)$

(Source: Authors' own elaboration)