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Research Article

Inclusion, equity, and diversity in task design at primary and lower secondary levels: The case of ratio and proportion

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ABSTRACT

This research employs a multiple case study to explore pre-service and in-service mathematics teachers' task design practice for addressing the principles of inclusion, equity, and diversity (IED) in the context of teaching ratio and proportion, and to observe the evolution of their practice upon participating in a workshop focused on integrating IED principles into task design. The participants were four in-service and two pre-service primary and lower secondary school teachers in Norway. Data was collected through semi-structured interviews with participants in a workshop and analyzed through the lens of the anthropological theory of the didactic. The findings suggest that participants modified their task design techniques during the workshop. Participants critically reflected on their practice, enabling them to amend their tasks to align with IED principles. These changes were detailed in the results, and their implications for task design practice were discussed.

Keywords: inclusion, equity, and diversity, task design, mathematics teachers, anthropological theory of the didactic, ratio, proportion

INTRODUCTION

Task design has been recognized as an important practice in mathematics education (Watson & Ohtani, 2015), in part because of its impact on students' learning (Christiansen & Walther, 1986; Doyle, 1983, 1988; Doyle & Sanford, 1985; Hiebert & Wearne, 1993). The importance of task design has also been reflected in numerous empirical studies over the last decade (e.g., Margolinas, 2013; Watson & Ohtani, 2015).

Teachers need to reflect on multiple aspects of their task design practice, often referred to as principles (Radmehr, 2023) or parameters (Watson, 2016) of task design. These might include curricular decisions, such as aligning tasks with the curriculum (e.g., Brown & Coles, 2013; Riard & Kaur, 2022), or pedagogical decisions about how tasks are planned and implemented (Sullivan et al., 2015). This process can be guided by reflecting on task design principles in mathematics education, particularly those related to the cognitive, affective, and social dimensions (Canogullari & Radmehr, 2026). One central aspect worth considering in task design is inclusion, equity, and diversity (IED), which encompasses several principles such as low-threshold, high-ceiling, and wide-wall (LTHCWW) tasks, as well as transparency (Canogullari & Radmehr, 2026).

IED is critical for enhancing students' motivation and engagement, as well as community building and a sense of belonging (Stephen et al., 2023). However, teachers face challenges in designing lessons or tasks that ensure IED. One common challenge acknowledged by several studies is teaching and catering to the needs of mixed-ability or heterogeneous student groups (Cheeseman & Kloger, 2018, 2021; Sullivan, 2007, 2015). This is due in part to the diversity of students' mathematical abilities and interests within these groups (Sullivan,

2007) and to differences in their learning approaches and paces (Cheeseman & Kloger, 2018). Teachers' practices can also contribute to this challenge. For instance, outlining expectations for low-achieving students could leave other students unfulfilled, and overly guiding them could lead to passive learning (Sullivan, 2015). This challenge has also manifested itself in task design practice. Research suggested that mathematics teachers need support in tailoring tasks to students' readiness (Sullivan et al., 2013).

Several teaching approaches have been proposed to meet the needs of learners in diverse learning groups, such as differentiated instruction (e.g., Tomlinson, 2017). Some practices have been found to be beneficial for ensuring equity in culturally diverse settings, such as the equitable teaching approach (Boaler & Staples, 2008), equitable mathematics instruction (Hand, 2012), and ambitious and equitable mathematics instruction (Chen, 2022).

Several empirical studies have focused on teachers' practices for inclusion and equity in mathematics classrooms across countries (e.g., Büscher & Prediger, 2024; Felton-Koestler, 2019; Mendez & van Es, 2025; Yılmaz et al., 2021), and their task design practices in diverse classrooms in the context of differentiated instruction (e.g., Bardy et al., 2021; Mellroth et al., 2021). These studies investigated an elementary school mathematics teacher's evolution in her views of equitable mathematics teaching (Felton-Koestler, 2019), mathematics teachers' perspectives on equity in remote instruction during the COVID-19 pandemic (Yılmaz et al., 2021), how secondary school (grade 5-grade 10) mathematics teachers integrate teaching demands for inclusive mathematics teaching (Büscher& Prediger, 2024), and how secondary school mathematics teachers' relational noticing supports fostering equity in the classroom (Mendez & van Es, 2025). Research on equity with pre-service teachers has also been examined from various perspectives over the last decade (Gilbert et al., 2025; Graham & Roth-McDuffie, 2023; Jackson & Jong, 2017; Lee & Herner-Patnode, 2025; McGraw et al., 2024; Moldavan & Gonzalez, 2023). These studies investigated elementary school pre-service teachers' perspectives on equity in mathematics teaching and learning through their written reflections (Jackson & Jong, 2017), how elementary and secondary school pre-service mathematics teachers identify and respond to inequalities in mathematics classrooms (Moldavan & Gonzalez, 2023), pre-service mathematics teachers' conceptions of equity (McGraw et al., 2024) and their perspectives and practices regarding equitable mathematics instruction for learners with diverse needs (Lee & Herner-Patnode, 2025).

Despite the number of teaching approaches that can foster inclusion and equity in diverse learner groups, there is limited empirical research that specifically examines how pre-service and in-service mathematics teachers design mathematical tasks to address IED in primary and lower-secondary mathematics classrooms. In other words, studies involving both pre-service and in-service teachers in the context of designing tasks that address the principles of IED appear to be lacking and are therefore needed to increase our knowledge in this area. Moreover, given the importance of addressing students' needs, it is essential to support both pre-service and in-service teachers in enhancing their practice. This need was also emphasized by Jackson et al. (2023), who argued that "existing lesson planning frameworks and design models do not explicitly foreground equity and inclusion within instructional planning, which are necessary in teaching every student" (p. 103).

This study aims to address this gap by providing pre-service and in-service mathematics teachers with a set of principles to design tasks that may help ensure inclusion and equity. To achieve this, we focus on one important component of the task design framework (i.e., IED), proposed by Canogullari and Radmehr (2026), as part of a larger project. Given the lack of practical guidance on task design to address IED, we treat this component as a central consideration. Furthermore, addressing IED principles in task design can allow us to examine their applicability across different communities and grade levels, and how they may vary in practice. We argue that participants' reflection on and engagement with this aspect can provide valuable insight into their experiences with IED in task design and reveal how these principles can be enacted in practice. Additionally, it may enable them to critically reflect on these principles in their task design practice.

Inclusion, Equity, and Diversity

Much has been written on inclusion (e.g., Faragher et al., 2016; Roos, 2019), equity (e.g., Boaler, 2022; Forgasz & Rivera, 2012; Hall et al., 2024; Hauk et al., 2021; Rogers & Kaiser, 1995; Vithal et al., 2024), and diversity (e.g., Bishop et al., 2015).

Inclusion is a "process that helps to overcome barriers limiting the presence, participation and achievement of learners" (United Nations Educational, Scientific and Cultural Organization [UNESCO], 2017, p. 7). It is an essential concept for promoting equity in education (Das, 2021) and is highly valued in mathematics education (Roos, 2019, 2023). Different perspectives have been offered for inclusion in mathematics education. For instance, Faragher et al. (2016) argued that "inclusive mathematics education acknowledges human diversity and involves supporting the diverse learning needs of all students in general mathematics classrooms" (p. 119). Although there are different views on its definition, inclusion in task design can be operationalized as considering students' knowledge and experiences (Radmehr, 2023).

The significance of equity is highlighted in the mathematics education literature (National Council of Teachers of Mathematics [NCTM], 2000), and it is closely linked to the notion of inclusion. Equity refers to "a concern with fairness, such that the education of all learners is seen as being of equal importance" (UNESCO, 2017, p. 7). Equity in mathematics education calls for high expectations for all learners and the belief that everyone can learn mathematics (NCTM, 2000). It implies understanding and making appropriate adjustments to teaching to meet the needs of diverse students, using resources such as curriculum materials and tools (NCTM, 2000). Phuong et al. (2017) defined equity as "creating an optimal learning environment that offers students opportunities to demonstrate their knowledge and improve their skills in various ways" (p. 11). Boaler (2013) considered equity as a way of acting and being in the classroom and how students relate to each other in terms of being respectful and fair, which she called relational equity.

Diversity is about "people's differences which may relate to their race, ethnicity, gender, sexual orientation, language, culture, religion, mental and physical ability, class, and immigration status" (UNESCO, 2017, p. 7). Addressing students' diverse needs is also an important consideration in mathematics education (Van de Walle et al., 2018).

Reflecting on the definitions of IED in the literature, we view these notions as related and unified rather than as isolated concepts in the context of task design. Our understanding of IED involves designing tasks that include all learners in classroom activities while accounting for their diversity, including but not limited to their backgrounds, experiences, interests, and learning levels. It is also worth noting that by "inclusion," we are not limiting our definition to addressing the needs of students with special needs; rather, we mean including all students in activities, recognizing that each student has unique needs that should be acknowledged and respected.

Ratios and Proportions

This study has chosen ratios and proportions as the mathematical subject of the tasks. These two notions are key mathematical concepts related to the development of proportional reasoning. Proportional reasoning, a form of mathematical reasoning (Cramer et al., 1989; Cramer & Post, 1993a), is a fundamental ability that students should develop in the early years of schooling (Small, 2015). It is a landmark in students' cognitive development (Cramer & Post, 1993b) and requires, among other things, the ability to understand the characteristics of proportional situations (Cramer & Post, 1993a). It is about "recognizing and forming multiplicative comparisons between quantities" (Small, 2015, p. 1). It has connections to many mathematical organizations, such as "scale, probability, percent, rate, trigonometry, the geometry of plane shapes, algebra, and fractions" (Dole & Shield, 2008, p. 19). Further examples include "similarity, relative growth and size, dilations, scaling, pi, constant rate of change, slope, speed, rates, percent, trigonometric ratios, probability, relative frequency, density, and direct and inverse variations" (Heinz et al., 2008, p. 528). Additionally, it has utility in other disciplines, including chemistry and physics, for concepts such as density, concentration, acceleration, and speed (Heller et al., 1989). Moreover, ratio and proportion are taught in both primary and lower secondary grade levels (Small, 2015). For these reasons, choosing ratio and proportion as a mathematical subject proved particularly relevant to this study. Given the above background, the following research questions (RQs) were addressed for this investigation:

1. What is the mathematical knowledge mobilized in the tasks designed by pre-service and in-service teachers, and how has this knowledge evolved after participating in a workshop on the principles of IED in task design for teaching ratio and proportion?

Table 1. Components of the paradidactic praxeology of task design in the context of this study

Component of the praxeology	Example for each component
Type of task (*T*)	Designing, adapting, and/or choosing tasks for teaching ratio and proportion
Technique (*τ*)	Approaches used to design, adapt, and or select a task
Technology (*θ*)	Principles underpinning task design approaches
Theory (*Θ*)	Teaching and learning theories in mathematics education that underpin the principles for task design

2. What is the nature of didactic knowledge involved in tasks designed by pre-service and in-service teachers, and how has this knowledge evolved after participating in a workshop on the principles of IED in task design for teaching ratio and proportion?

Anthropological Theory of the Didactic

The Anthropological Theory of the Didactic (ATD), proposed by Chevallard (2019), has emerged as a research paradigm in French didactics of mathematics (Bosch et al., 2019). This study employs the two main notions of the ATD—praxeology and reference model. In what follows, these notions are described, and their application is given.

Praxeology

According to ATD, every human activity can be viewed as a praxeology, consisting of a praxis and a logos block (Chevallard, 2019). Praxis block (*Π*) involves a type of task (*T*) and technique(s) (*τ*), while logos block (*Λ*) involves technologie(s) (*θ*) and theorie(s) (*Θ*) (Chevallard, 2019). Accordingly, techniques explain how to perform a specific type of task. Technologies explain and justify the techniques employed, and theories justify these technologies. A praxeology is a union of praxis and logos blocks, which can be denoted as "$\Pi \oplus \Lambda = [T / \tau] \oplus [\theta / \Theta] = [T / \tau / \theta / \Theta]$" (Chevallard, 2019, p. 92).

Different types of praxeologies exist, e.g., mathematical, didactic, and paradidactic (Miyakawa & Winsløw, 2013). Mathematical praxeologies are about the knowledge of mathematics, didactic praxeologies are concerned with that of teaching, and paradidactic praxeologies relate to the preparation and evaluation of teaching activities (Miyakawa & Winsløw, 2013). Given that task design is primarily related to preparation for teaching and occurs before classroom teaching, this study views task design as a paradidactic praxeology, whose main task is to design, adapt, and/or choose tasks for teaching ratio and proportion (see **Table 1**).

Reference model

ATD designs and employs reference models for the praxeologies of knowledge to be analyzed (Gascón & Nicolás, 2022). One such model is the epistemological reference model (ERM), which explains what to study (Gascón & Nicolás, 2022). There are also praxeological reference models used to analyze the praxis and logos blocks of praxeologies, such as those found in textbooks (e.g., Wijayanti & Winsløw, 2017). These models are viewed as heuristic and provisional tools that are designed relative to the didactic problem at hand (Gascón & Nicolás, 2022).

This work employs two types of reference models, a paradidactic praxeological reference model (PPRM) and an ERM. The PPRM was used to investigate pre-service and in-service mathematics teachers' task design praxeology in terms of its praxis and logos block and observe how this praxeology has evolved after attending a workshop. The ERM was used to investigate the mathematical praxeologies mobilized in their task, focusing on their structure, functioning, and utility. Below, the two models are outlined, and their roles in this study are explained.

A Paradidactic Praxeological Reference Model for Task Design

This study uses one aspect of the task design framework, IED, developed by Canogullari and Radmehr (2026), as a PPRM for task design. This type of reference model was developed to explore teachers' reflective practices and has been shown to be potentially helpful for this purpose (e.g., Hakamata, 2021). Therefore, it is found suitable to use this type of model, given the nature of praxeology under investigation.

The IED aspect of the task design framework developed by Canogullari and Radmehr (2026) involves three principles:

Table 2. A paradidactic praxeological model for task design in relation to the IED aspect

Component of the praxeology	Example for each component
Type of task (*T*)	Designing, adapting, and/or choosing tasks for teaching ratio and proportion
Technique (τ)	τ_1: Creating sub-tasks with increasing cognitive difficulty τ_2: Using open problems allowing multiple solution paths and answers τ_3: Designing a task with a realistic, authentic, and culturally relevant context
Technology (θ)	θ_1: Low-threshold, high-ceiling, and wide-wall θ_2: Meaningful context
Theory (Θ)	Theories of learning in mathematics that underpin participants' task design (e.g., RME, constructivism, and sociocultural theories)

(1) low-threshold, high-ceiling, and wide-walls,

(2) meaningful context, and

(3) transparency.

This study focuses on the first two of these principles, considering them as a technology ($\theta_{1,2}$) within the logos block of the task design praxeology for the given task type T_1, i.e., designing, adapting, and/or selecting tasks for teaching ratio and proportion (see **Table 2**). The theory (Θ) was not predetermined but referred to theories of teaching and learning that could be used to design tasks, such as realistic mathematics education (RME), constructivism, or sociocultural theories. The praxis block consisted of three techniques and two technologies that supported them. The first technique (τ_1) involved creating sub-tasks with increasing cognitive difficulty, illustrating how to address low-threshold and high-ceiling principles. The second technique (τ_2) involved using open problems that allowed multiple strategies and answers. The third technique (τ_3) was designing a task with a realistic, authentic, and culturally relevant context.

Principle 1. Low-threshold, high-ceiling, and wide-walls

The concept of "low-threshold, high-ceiling" was first considered in the context of designing software tools (e.g., Myers et al., 2000). Later, the "wide-wall" feature was suggested to enhance the tool's creativity for its users (e.g., Resnick et al., 2005). In mathematics education, the terms "low-threshold, high-ceiling" and "wide-wall" describe tasks that are accessible yet challenging and open-ended (Boaler, 2015). They can also be viewed as principles for ensuring inclusion in task design (Radmehr, 2023).

Low-threshold tasks are accessible to students with little prior knowledge, and high-ceiling tasks offer students the opportunity to advance their mathematical thinking beyond their current level (Gadanidis & Hudges, 2011). Not only are they considered more interesting and engaging, but also, they can be used to foster creativity (Boaler, 2015), curiosity, and imagination (Gadanidis & Hudges, 2011). Most importantly, students with different achievement levels can engage with these types of tasks (Boaler, 2015). Overall, the use of such tasks is supported by other researchers for their potential benefit to all learners (e.g., Papadopoulos, 2020).

In addition to having a low-threshold and a high-ceiling, the task can also have a wide-wall. In the context of programming, a wide-wall is viewed as a principle that supports exploration and creativity (e.g., Resnick et al., 2005). In mathematics, Gadanidis et al. (2017) viewed a wide-wall as a feature providing students with "a wider audience for their mathematical thinking, beyond their teacher and classroom peers" (p. 91). In task design, a wide-wall can suggest that a task can be solved in various ways (Radmehr, 2023). Taking a holistic view, we perceive a wide-wall as a principle characterizing tasks that can be approached in many different ways, including those with multiple solutions.

Principle 2. Meaningful context

Meaningful context is one of the key principles in task design in mathematics education (Canogullari & Radmehr, 2026; Radmehr, 2023). One way to define context is to view it as a situation embedded in a problem, whether real or imaginary (Vos, 2020). From the perspective of the RME, contextual problems are defined as those that are experientially real to students, and even pure mathematical problems are seen as contextual on the same grounds (Gravemeijer & Doorman, 1999). However, simply adding more descriptions to the story of a problem does not make it more contextual; rather, problems are contextual to the extent that they provide meaningful experiences for the students (Roth, 1996).

The context of a mathematical task can differ in terms of its relation to reality (Vos, 2020). According to Vos (2020), the context of a mathematical task can be in five different forms:

(1) a bare task without any real-world context,

(2) a task with a mathematical context,

(3) a dressed-up context,

(4) a realistic context, and

(5) an authentic context.

A bare task consists merely of mathematical language without any context to which the mathematical objects in the task are linked (Vos, 2020). A task with a mathematical context includes descriptions that provide meaning to mathematical concepts (Vos, 2020). These types of tasks "are about mathematical objects and their properties," which could sometimes be linked to real-world objects (Vos, 2020, p. 39). Dressed-up tasks include realistic contexts, but these contexts do not justify the need to solve the problem (Vos, 2020). A realistic task is one in which the context justifies the need to solve it (Vos, 2020). This type of task can involve a scenario that students can experience in real life or one they can imagine (Vos, 2020). Authentic tasks are those "in which the origin of the context is explained through convincing resources. In this category, also, the context justifies the question, and an answer is useful within the described context" (Vos, 2020, p. 40). Importantly, "authenticity is a characteristic that requires clear evidence, for example, through photos (as opposed to drawings), or when governmental datasets are used in a statistical task" (Vos, 2020, p. 40).

The context of the task can also be culturally responsive (Gallivan, 2017). This type of task is "embedded into a context that draws upon students' (or one student's) cultural and/or home and community funds of knowledge" (Gallivan, 2017, p. 95). We consider cultural responsiveness as a complementary feature for tasks. By this, we mean that a task can be realistic or authentic without necessarily being culturally responsive or relevant to students' experiences. For instance, a task can involve a realistic shopping context in the US, using US Dollars as the currency. The problem could include authentic pictures. However, the dollar prices given in the problem context may not be culturally relevant to students living in Scandinavian countries.

An Epistemological Reference Model for Ratio and Proportion

One way to build an epistemological model of a mathematical object is to study its structure, functioning, and utility (Chevallard, 2024). Structure explains what an object is made of, b) functioning explaining how an object works, and c) utility explaining what an object is used for (Chevallard, 2024). Drawing on the literature, this study develops an epistemological model of the mathematical objects of ratio and proportion. It is then used to examine the mathematical praxeologies of the tasks that participants created.

Structure

According to Chevallard (2024), the structure of any mathematical object pertains to its components. Here, as part of the structure of ratios and proportions, we present our conceptualization of their definitions and properties (see **Figure 1**). Ratio is a (multiplicative) comparison of two quantities (Behr et al., 2006), and proportion is an equality of two ratios (Behr et al., 1983; Ben-Chaim et al., 1998; Heller et al., 1989). Measure space, a term coined by Vergnaud, is a model used to explain the multiplicative relationship in proportional situations (Cramer & Post, 1993b). If two ratios are said to be in the same measure space, then the relationship between them is considered within ratio, whereas if they are corresponding quantities in two different measure spaces, then their relation is called between ratio (Van de Walle et al., 2018). The numerical values of the ratios can also be integers or non-integers (Cramer & Post, 1993b). **Figure 1** presents a possible way to structure the ratio and proportion at the primary and secondary school levels, including their definitions and properties.

Functioning

According to Chevallard (2024), the functioning of a mathematical object explains how it works. Ratio and proportions can be applied in various mathematical contexts and have different interpretations depending on their usage. In mathematical problems, ratios can be used in four types of situations: part-to-part ratios, part-to-whole ratios (e.g., fractions and percentages), ratios as quotients, and ratios as rates (Van de Walle et

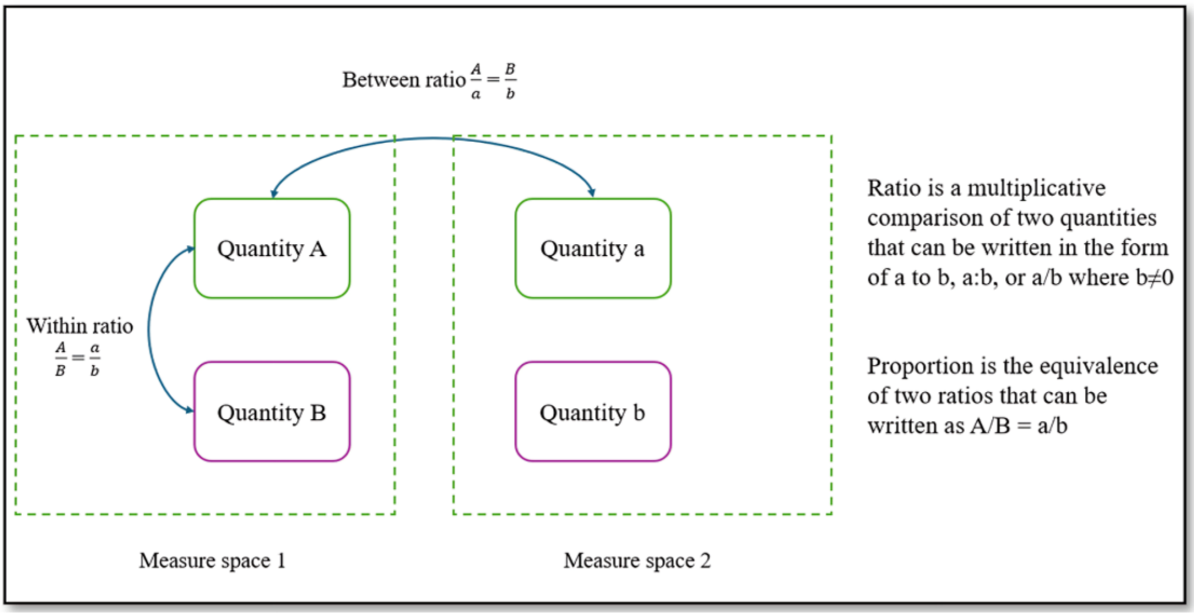

Figure 1. Structure of ratio and proportion (adapted from Van de Walle et al., 2018)

al., 2018). The ratio of 5 cats to 3 dogs in a pet store can be an example of a part-to-part ratio. In the same scenario, the ratio of cats to all pets is a part-to-whole ratio and can also be expressed as a fraction or a decimal. Ratio can also be treated as a quotient. An example of a ratio as a quotient is calculating the unit price of a product (e.g., 2 chocolate bars cost 35 NOK; what is the unit price of 1 chocolate bar?). In this scenario, the ratio is used as a quotient. A ratio, as a rate, requires two different units and their comparison (Van de Walle et al., 2018). An example of a ratio as a rate could be a car traveling 120 km per hour.

Proportions can also be applied to various problem situations. These can include missing-value problems, numerical-comparison problems, and qualitative-prediction and comparison problems (Cramer & Post, 1993a). In missing value problems, three of the quantities are known, and the goal is to find the missing one (Cramer & Post, 1993b). An example of a missing-value problem is: If a bag of apples weighs 2 kg, how much do two bags of apples weigh? In numerical comparison problems, all quantities are known, and the goal is to compare the ratios (Cramer & Post, 1993a). An example of a numerical comparison problem is: Brand A is a 5-liter detergent that can perform 30 washes, while brand B is a 3-liter detergent that can perform 15 washes. Which one can do more washes per liter? In qualitative prediction and comparison problems, comparison does not rely on numbers, but it still requires an understanding of proportions (Cramer & Post, 1993b). An example qualitative comparison problem could be: Erik and Sue want to drink coffee. Erik prefers a smaller cup and adds more sugar/milk, while Sue prefers a bigger cup and adds less sugar/milk. Which coffee tastes stronger? In the same scenario, a qualitative prediction problem could be: if Sue changes the size of her cup and orders a bigger cup of coffee with the same amount of sugar/milk, would it taste stronger/weaker/the same?

Utility

According to Chevallard (2024), the utility of any mathematical object concerns what the object is used for. This can include an object's utility inside mathematics (i.e., intra-mathematical utility) and its utility outside mathematics (i.e., extra-mathematical utility) (Topphol, 2023). **Table 3** shows some examples of the intra-mathematical and extra-mathematical utility of ratio and proportion.

Given the two main models above, the current study aims to investigate the task design praxeology of in-service and pre-service mathematics teachers and how it has evolved following a workshop. The following RQs were addressed for this purpose:

1. What mathematical praxeologies are mobilized in the tasks designed by pre-service and in-service teachers in terms of structure, functioning, and utility, and how has this knowledge evolved after participating in a workshop on the principles of IED in task design for teaching ratio and proportion?

Table 3. Some examples of the intra and extra mathematical utility of ratio and proportion

Intra-mathematical utility	Extra-mathematical utility
Numbers and arithmetic: multiplication and division problems, fractions, decimals, and percents	Cooking and baking: Finding the amount of ingredients in a recipe Shopping: Calculating the unit price of a product
Algebra: slope of a function	Architecture and infrastructure: Calculating the incline of roads and hills Chemistry and physics: Calculating chemical reaction rates, density, speed, and acceleration
Geometry and measurement: similarity	Cartography: Design of maps Photography: Enlarging or shrinking a photo captured by a camera
Probability: likelihood of events	Games and sports: Calculating the likelihood of a specific outcome when drawing a card

2. What is the nature of paradidactic knowledge involved in tasks designed by pre-service and in-service teachers in terms of praxis and logos block of the PPRM, and how has this praxeology evolved after participating in a workshop on the principles of IED in task design for teaching ratio and proportion?

Practices for Addressing Inclusion, Equity, and Diversity in Mathematics Classrooms

Several pedagogical practices have been used to address IED in mathematics classrooms, such as complex instruction (Cohen, 1998; Cohen et al., 1995, 1999; Lotan, 2025; Lotan et al., 1994), equitable teaching approach (Boaler & Staples, 2008), equitable mathematics instruction (Hand, 2012), adaptive equity-oriented pedagogy (Phuong et al, 2017), and ambitious and equitable mathematics instruction (Chen, 2022).

One of the earliest examples of equity-promoting pedagogies, known as complex instruction, emerged in the late 1970s and is suitable for heterogeneous student groups, promoting equitable teaching for all learners by having students work in groups on open-ended tasks (Cohen, 1998). This pedagogical approach emphasizes conceptual learning and aims to promote higher-order thinking skills by engaging learners in challenging tasks (Lotan et al., 1994). One of the teachers' main roles is to ensure equal participation among students (Cohen et al., 1999), and students draw on one another as resources (Cohen, 1998; Cohen et al., 1995, 1999; Lotan et al., 1994).

This pedagogical approach has been tested empirically by other researchers (e.g., Boaler & Staples, 2008). In their study, Boaler and Staples (2008) worked with 700 students from three high schools, representing diverse social backgrounds and cultures, over five years. One of the schools in their study, with the pseudonym Railside school, used a blend of reform-oriented curriculum and complex instruction that offered everyone equitable opportunities, while the other two schools used a mix of traditional teaching and interactive mathematics programs that focused primarily on problem-solving. Results show that students taught with the reform-oriented curriculum and complex instruction outperformed those in the other two schools, enjoyed learning more, and that the achievement gap between ethnic groups in Railside school was reduced.

Another equity-oriented pedagogy was suggested by Phuong et al. (2017), who developed and implemented an intervention with college students. This approach, named adaptive equity-oriented pedagogy, integrates various instructional strategies, including critical and democratic teaching, assessment-focused methods, strength-based approaches, game-based learning, and multimodal techniques. Researchers discovered that this form of intervention led to positive results, including increased engagement, participation, collaboration, sense of community, and academic performance.

There are also approaches particularly suited to teaching in culturally diverse settings, such as culturally responsive pedagogy (Irvine, 1990), culturally relevant pedagogy (Ladson-Billings, 1995a, 1995b), and culturally responsive teaching (Gay, 2000, 2002, 2015). For instance, Irvine (1990) suggested that teacher education programs should train teachers to use culturally responsive pedagogy and listed 10 key areas in which teachers should be equipped. Among these is gaining a better understanding of and appreciation for students' diverse cultural backgrounds and tapping into this knowledge to adjust teaching (Irvine, 1990). According to Ladson-Billings (1995b), culturally relevant pedagogy aims to increase students' academic achievement, cultural competence, and critical-thinking abilities, inviting them to question the norms and values of cultures and institutions. Moreover, Turner et al. (2022) developed a tool for observing classroom activities that involves culturally responsive and equity-oriented teaching of mathematical modeling in grade 3-grade 5.

Several strategies have been recommended for structuring mathematical activities in a way that promotes equity, such as incorporating challenging content, promoting group work, and fostering deep thinking (Boaler, 2022) or building on students' experiences and using rich tasks with multiple entry points that involve varying levels of difficulty, often known as low-threshold and high-ceiling tasks (Chen, 2022). Additionally, a few studies have discussed inclusion more specifically in the context of task design (e.g., Büscher, 2019; Calleja, 2024; Healy et al., 2013; Radmehr, 2023). For instance, Healy et al. (2013) created learning scenarios that used multimodal representations of mathematical ideas (e.g., texture, sound, and color), not only to respect and meet the needs of students with diverse sensory (dis)abilities but also to provide a path to think about mathematics in different ways. Calleja (2024) used the principles of universal design for learning to characterize tasks that foster inquiry and inclusion in mathematics classrooms, such as tasks with multiple entry points, that increase students' participation, that provide access to a wide range of mathematical concepts, and that invite productive struggle. Büscher (2019) investigated how teachers use the idea of differentiating tasks by content and access to make mathematics more inclusive. Overall, studies show that IED are valuable considerations when designing classroom activities that address diverse needs.

Research on LTHCWW and Meaningful Context in Task Design

There are relatively few studies that address the principles of LTHCWW and meaningful context in task design. These studies have a variety of research goals and employ various methodologies with participants from diverse backgrounds, such as students (e.g., Gadanidis et al., 2017; Gjesteland & Vos, 2019), mathematics teachers (Stehr et al., 2018), and a group of professionals including teachers, teacher educators, mathematicians, and statisticians (e.g., Doorman et al., 2019).

For instance, Gadanidis et al. (2017) explored the potential of computational thinking with first-grade students in the context of geometry and probability tasks through a classroom case organized around seven pedagogical affordances related to computational thinking, one of which was the use of LTHCWW tasks. In another study, Stehr et al. (2018) investigated how technology use by grade 3-grade 5 mathematics teachers in the classroom supported low-threshold, high-ceiling tasks and productive discussion. More recently, Doorman et al. (2019) interviewed two design teams consisting of different professionals and reported that meaningful context, low-floor, and high-ceiling are important task characteristics considered by both teams. In another study, Gjesteland and Vos (2019) worked with first-year engineering students. They investigated their affect, operationalized as challenge and flow, while students worked on an inquiry-based task with low-threshold, high-ceiling characteristics. Their results indicated that students experienced the challenge and flow to a great extent. The use of meaningful context was also found beneficial in studies such as Sawatzki (2017). Sawatzki (2017) reported that in her design-based study across four schools involving teachers and grade 5-grade 6 students, both groups gained notable benefits from engaging with authentic problems because the problems were relevant, accessible, and useful.

Although these studies address the principles of LTHCWW and meaningful context, the primary goal was not to ensure IED, highlighting the need to address these principles in the context of task design further.

METHODOLOGY

The current research employs a multiple-case study design (Yin, 2018) with two cases. Case 1 comprises four in-service mathematics teachers working in primary and lower-secondary schools in Norway. In comparison, case 2 includes two pre-service mathematics teachers in the final year of their master's program in primary and lower secondary teacher education for year 5-year 10 in one of the public universities in Norway (see **Table 4**). All participants were recruited based on their convenience and availability.

Data Collection: Workshop

Participants were invited to one-on-one interviews and to attend a workshop conducted by the first author on the IED aspect of the task design framework (see Canogullari & Radmehr, 2026). Before the interviews, participants were asked to bring two tasks, one bare task and one contextual task (Vos, 2020), so they could

Table 4. Participant information

	Participant	Gender	The grade they teach	Teaching experience
Case 1. In-service mathematics teachers	Nora	Female	Grade 1-grade 10	2 years
	Lars	Male	Grade 7-grade 10	19 years
	Olivia	Female	Grade 2-grade 9	20 years
	Elin	Female	Grade 8-grade 10	25 years
Case 2. Pre-service mathematics teachers	Tor	Male	Grade 2-grade 10	2 years as a substitute teacher
	Frida	Female	Grade 1-grade 10	4 years as a substitute teacher

reflect on and revise them during the workshop. This also enabled us to see whether they considered these principles when selecting tasks, without being exposed to them beforehand. The tasks participants were asked to bring to the interviews were intended to serve as complementary data sources to enrich the interview results.

The interviews began with general questions, such as "How do you design, adapt, or select tasks for teaching ratio and proportion?" This allowed us to explore teachers' task design praxeology in general. Following this phase, the first author conducted a workshop that presented the principles of IED in task design. This workshop was structured around two main activities.

In Activity 1, the first technology and its corresponding techniques in the PPRM were presented to the participants. In other words, the principles of low-threshold, high-ceiling, and wide-walls were defined, and examples for each were provided. Following this, teachers were asked to reflect on their bare task, and were asked the following questions: "Does your task reflect the features of LTHCWW? If so, how? If not, how would you refine your task to better align with LTHCWW principles?" In this activity, participants were specifically asked to reflect on the bare task to help them focus primarily on the principles of LTHCWW without the potential distraction of the task's contextual features.

In Activity 2, the second technology and its corresponding technique in the PPRM were presented to the participants. In other words, the principle of meaningful context in task design was defined, and participants were shown example tasks for each context type. Five different types of contexts (Vos, 2020) and their relation to real life were presented. Following this, teachers were asked to reflect on their contextual task. They were asked the following questions: "Does your task reflect the features of realistic, authentic, culturally responsive, and transparent? If so, how? If not, how would you refine your task to better align with these principles?" These questions invited teachers to reflect on their tasks and open the door to making revisions if they deemed it necessary.

In both activities, participants were asked to revise their tasks if they believed they were not in line with the given principles. They were also told that if they did not want to formulate a new task, they could suggest ways to improve their task in alignment with these principles. Overall, these activities aimed to investigate whether there were any improvements or shifts in teachers' task design praxeology after participating in a workshop focused on the principles of IED in task design. Each participant was interviewed once in a single session lasting approximately 60-90 minutes, conducted either in person or online via Zoom, depending on their preference. **Table 5** shows the content of the workshop.

Participants' written consent was obtained through an informed consent form, which provided them with information about the goal of the study and assured them of the confidentiality of the data they would provide. Moreover, participant names were replaced with pseudonyms to protect their confidentiality.

Data Analysis

Data consisted of participants' tasks and interview transcripts. An inductive approach (Creswell & Creswell, 2018) was initially employed to analyze participants' tasks and interviews, and themes were subsequently derived from the techniques they used. This process began with inductive coding of the initial part of the interviews, in which participants were asked, "How do you design, adapt, or select asks for teaching ratio and proportion?" Interview transcripts provided us with information about participants' praxis and logos in task design praxeology. Themes emerged regarding the characteristics of their responses (e.g., how they design tasks and why they approach them in particular ways).

Table 5. Content of the workshop

Slide number	Content of the slide	Relevant literature	AD
Slide 1-2	Definitions of IED in task design	(NCTM, 2000; UNESCO, 2017)	5 minutes
Slide 3-4	Importance of addressing IED in task design and its three principles	(Canogullari & Radmehr, 2026; Chevallard & Bosch, 2020; Geiger, 2019; Radmehr, 2023; Vos, 2020)	5 minutes
Slides 5-7	Definition of LTHCWW principles, and an example task addressing LTHCWW	(Canogullari & Radmehr, 2026; Boaler, 2013; Gadanidis & Hudges, 2011; Lamon, 2020; NCTM, 2000; Radmehr, 2023; Van de Walle et al., 2019)	15 minutes
Slide 8	Activity 1	No reference	15 minutes
Slide 9-15	Definition of the meaningful context principle, types of contexts, and example tasks for each type	(Gallivan, 2017; Vos, 2020)	15 minutes
Slide 16	Definition of the transparency principle	(Borromeo Feri, 2007; Burkhardt & Swan, 2013; Geiger, 2019)	5 minutes
Slide 17	Activity 2	No reference	15 minutes

Note. AD: Approximate duration

Then, this was followed by a more deductive approach (Creswell & Creswell, 2018), in which participants' responses were categorized into the praxis and logos aspects of task design praxeology. Participants' pre- and post-workshop tasks, as well as their reflections on these tasks, were analyzed in reference to PPRM and ERM. The PPRM enabled us to investigate how participants applied the principles of IED in task design. We also examined whether participants used techniques beyond those presented to them. Additionally, the ERM allowed us to explore the structure, functioning, and utility of the mathematical praxeologies mobilized in their tasks.

Regarding PPRM, only the first IED principle (i.e., LTHCWW) was analyzed for Activity 1, as we specifically asked participants to bring a bare task to avoid distraction from the task's contextual features. Only the second IED principle (i.e., meaningful context) was analyzed for Activity 2; our primary focus was on the task's contextual aspects. To analyze the contextual aspects of the task, Vos' (2020) framework was employed, along with a focus on cultural responsiveness.

Moreover, several approaches have been employed to enhance reliability, including conducting regular meetings to discuss the overall research process, utilizing triangulation by combining multiple data sources such as interview transcripts and participants' tasks, and providing a thick description of the cases and the research as a whole (Creswell & Poth, 2018).

FINDINGS

The current study explored pre-service and in-service teachers' task design praxeology and its evolution following participation in a workshop on the principles of IED for task design in teaching ratio and proportion. The results of both activities are presented below.

Results of Activity 1: Analysis of Teachers' Bare Tasks Based on PPRM and ERM

Participants' original and revised tasks were analyzed using the PPRM and ERM to examine how their praxeologies evolved (see **Table 3**). Half of the participants brought a low-threshold task (Elin, Olivia, and Nora). This meant that their tasks were easy for everyone at the targeted grade level to start with. For instance, Nora's task was to ask about the relationship between 24 and 4, which can be considered a low-threshold task for sixth graders.

However, more than half of the original tasks could be characterized as having a high-ceiling (Lars, Elin, Tor, and Frida), meaning they involved questions that were challenging for at least some of the students for whom they were designed. Among them, Elin's task involved several questions of varying difficulty, making it both low-threshold and high-ceiling.

After Activity 1, all participants' tasks, except Olivia's, whether original or revised, were categorized as having a high-ceiling, indicating a change in participants' task design praxeology. For instance, Nora's initial task was a low-threshold task for sixth graders. However, after Activity 1, she was able to modify it to align with the high-ceiling principle. Nora's original task was: "Find the relationship between 24 and 4. Answer: 24

Table 6. Analysis of tasks in Activity 1

			Original task		Revised task	
			In-service teachers	Pre-service teachers	In-service teachers	Pre-service teachers
PPRM	Low-threshold		Elin, Olivia, Nora	-	Elin, Olivia, Nora	-
	High-ceiling		Lars, Elin	Tor, Frida	Lars, Elin, Nora	Tor, Frida
	Wide-walls		Lars, Elin	Tor	Lars, Elin, Olivia	Tor
ERM	Structure		Ratio (Olivia, Nora)	Ratio and proportion (Tor, Frida)	Ratio (Olivia, Nora)	Ratio and proportion (Tor, Frida)
	Functioning		Part-to-whole ratio (Olivia), ratio as a quotient (Nora)	Part-to-part ratio (Tor, Frida), missing value (Tor, Frida)	Part-to-whole ratio (Olivia), ratio as a quotient (Nora)	Part-to-part ratio (Tor, Frida), part-to-whole ratio (Tor), missing value (Tor, Frida)
	Utility	Intra mathematical utility	Algebra (Lars, Elin), numbers (Olivia, Nora)	Numbers (Tor), geometry and measurement (Frida)	Algebra (Lars, Elin) Numbers (Olivia, Nora)	Numbers (Tor), geometry and measurement (Frida)
		Extra mathematical utility	Identifying the number of jellybeans in a bag (Olivia)	-	Identifying the number of jellybeans in a bag (Olivia)	Making lemon juice (Tor)

Note. Lars, Elin, and Frida reflected on their task but did not formulate a new one, whereas Olivia and Nora reflected and made suggestions as to how their tasks could be improved, aligning with the LTHCWW principle (Only Tor formulated a new task. Lars' and Elin's task was not related to ratio and proportion, therefore, they were not analyzed in terms of structure and functioning. However, their utility was analyzed and noted)

is […] times bigger than 4". Nora's technique for making her task align with a high-ceiling was to change the numbers in the given ratio. Her initial task involved numbers that were easily divisible, which kept cognitive demand low. She changed those numbers with those that were not easily divisible by each other.

> Nora: So, I do not think that I followed the low-threshold, high-ceiling, wide-wall principles. I chose numbers that are easy for students […additionally,] we can make it harder by using the numbers 10 to 4, for example. […] using bigger numbers outside of the small multiplication table [that] they know.

Another example is Tor's task, which was: "The relationship between two numbers is 3 to 5. The biggest number is 40. What is the smaller number?" Tor's technique for making his task both low-threshold and high-ceiling included putting his problem into a realistic context and then changing the numbers to adjust the difficulty of his task:

> Tor: If I were to give it to 6th grade […], I would not just give them this task and expect them to solve it. I would have to be there with them […], you can give an example of it, like a situation they can relate to in this relationship 3 to 5 […]. The most obvious example is in Norway […], you make saft […] You have this relationship. And try to make sense of it. Let us say you need 40 liters of saft or something. Maybe that would help them because this is not a low-threshold task […], if I were to do that, I would change the numbers because 5 is not very realistic. No saft is in that relationship […], "The lemon juice is mixed in a ratio of 1:6. How much juice do you need to make 4 liters of mixed juice for the sports day?"

Regarding wide walls, three participants brought tasks addressing this principle (Lars, Elin, and Tor). During Activity 1, only one participant (Olivia) amended her task to meet this criterion, while the remaining two did not do so (Lars and Frida). Olivia's technique for aligning her task with this principle was to ask students to solve it in several different ways. Table 6 shows the analysis of tasks in Activity 1.

Regarding their structure, two in-service teachers brought a ratio task (Nora and Olivia), and two pre-service teachers brought tasks involving ratio and proportion (Tor and Frida). After Activity 1, there were no changes in the structure of their tasks.

In terms of their functioning, one teacher used a part-to-whole ratio (Olivia), another used a quotient (Nora), and pre-service teachers used part-to-part ratios (Tor and Frida) in their initial tasks. Pre-service teachers' tasks were also classified as missing-value problems because they involved proportional situations requiring the identification of the missing quantity when three others were known. After Activity 1, there were

Table 7. Teachers' practice on their contextual task before and after revision during the workshop

			Original task		Revised task	
			In-service teachers	Pre-service teachers	In-service teachers	Pre-service teachers
PPRM	A task with a mathematical context		-	-	-	-
	Dressed-up task		Lars, Olivia	Frida	-	-
	Realistic		-	Tor	-	Frida
	Authentic		Elin, Nora	-	Olivia, Lars, Elin, Nora	Tor
	Culturally responsive		Lars, Elin, Olivia, Nora	Tor, Frida	Lars, Elin, Olivia, Nora	Tor, Frida
ERM	Structure		Ratio (Olivia, Nora, Elin), proportion (Nora)	Ratio (Frida, Tor), proportion (Tor)	Ratio (Olivia, Nora, Elin), proportion (Nora)	Ratio (Frida, Tor), proportion (Tor)
	Functioning		Part-to-part ratio (Olivia), part-to-whole ratio (Nora, Elin), missing value (Nora)	Part-to-part ratio (Tor, Frida), part-to-whole ratio (Tor, Frida), numerical comparison (Tor)	Part-to-part ratio (Olivia), part-to-whole ratio (Nora, Elin), missing value (Nora)	Part-to-part ratio (Tor, Frida), part-to-whole ratio (Tor, Frida), numerical comparison (Tor)
	Utility	Intra mathematical utility	Algebra (Lars), probability (Elin), numbers (Nora, Olivia)	Numbers (Tor, Frida)	Algebra (Lars), probability (Elin), numbers (Nora, Olivia)	Numbers (Tor, Frida)
		Extra mathematical utility	Grocery shopping (Lars), recycling paper (Olivia), chance game (Elin), making pancakes (Nora)	Comparing prices (Tor), making juice (Frida)	Grocery shopping (Lars), recycling paper (Olivia), chance game (Elin), making pancakes (Nora)	Comparing prices (Tor), making juice (Frida)

Note. Participants' tasks were analyzed based on the five types of contexts proposed by Vos (2020); however, the first type of context, the bare task, was excluded from this analysis because participants were specifically asked to bring contextual tasks for Activity 2 (Cultural responsiveness was also analyzed in participants' tasks, as well as in the contexts in which they were embedded. Lars' task was not analyzed in terms of structure and functioning because it was not related to ratio and proportion. However, its utility was analyzed and noted)

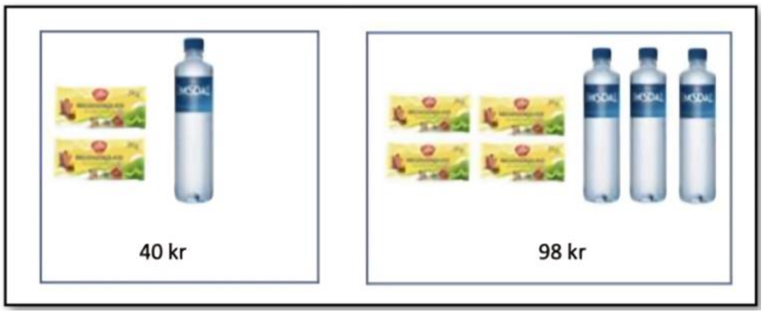

Figure 2. Lars' original task before Activity 2 (Source: This figure displays the participant's original task and is shared with the permission of the participant)

no changes in the tasks of two in-service teachers (Nora and Olivia) and one pre-service teacher (Frida), while one pre-service teacher changed the functioning of his task by introducing a part-to-whole ratio (Tor).

Concerning their utility, all participants' original tasks had an intra-mathematical utility in domains such as algebra (Lars and Elin), numbers (Olivia, Nora, and Tor), and geometry and measurement (Frida). Only Olivia's original task contained extra mathematical utility, with a daily-life situation. No changes were observed in any of the tasks after Activity 1 in terms of intra-mathematical utility. However, Tor's revised task had a utility in a daily-life context, i.e., making lemonade.

Results of Activity 2: Analysis Based on PPRM and ERM

Participants' initial and revised tasks were analyzed based on the PPRM and ERM (see **Table 7**). In terms of the meaningful context principle, there was no task classified as a task with a mathematical context, neither at the beginning nor after the revision.

Question 2

Pupils on the Eco Committee in Year 6 wanted to investigate how many worksheets were being printed each week. They found that there were 160 maths worksheets and 80 English worksheets. What is the ratio of maths to English worksheets?

Figure 3. Olivia's original task before Activity 2 (This figure shows the author's updated font style version of the original task given by Olivia. She retrieved the task from https://thirdspacelearning.com/blog/ratio-word-problems/)

A ratio indicates how the juice should be mixed, so it is neither too strong nor too weak.

- 1 L of concentrated juice from MO%RO sukker.
  Mixing ratio 1 : 3
  Price: 19.60 kr
- 1 L of concentrated juice from BringeBærum
  Mixing ratio 1:4
  Price: 24.90 kr

a) What is the price per liter of juice at BringeBærum and MO%ROsukker?

b) Which of the types of juice is the cheapest when you look at the amount of juice per krone?

c) How much cheaper is one type of juice in percent?

Figure 4. Tor's original task before Activity 2 (This figure displays the translated version of the participant's task, originally in Norwegian. It is translated by the authors)

Half of the original tasks were characterized as dressed-up tasks (Lars, Olivia, and Frida) due to a lack of justification for solving the problem, despite one of them featuring an authentic picture (Lars). For instance, Lars brought the task in **Figure 2** for this activity.

After Activity 2, participants successfully made their tasks realistic (Frida) or authentic (Olivia and Lars) by employing various techniques. For instance, Frida's technique for making her task more realistic involved bringing concrete materials to work with, such as bringing cups to the classroom for mixing different ratios. This way would add a justification for solving her task, making it more realistic. Another example was Olivia's task (see **Figure 3**), which she suggested could be made more realistic or authentic by working on the context of the problem.

> Olivia: As it stands, as a dressed-up task, but it would be very easy to make it into a realistic or an authentic task if we were to modify it. So, taking the context, which is a great context, here is an eco-club at your school. In Norway, this is especially relevant because having the green flag for your school is a point of pride [...] modifying the context in the question so that it says: The eco club has a goal of reducing paper waste [...] So here is that data that makes it authentic according to that which Vos said because it has additional data [...] How much should Math reduce and how much should English reduce to meet that particular goal? and I think that it makes it an authentic task to work with [...].

Only one of the original tasks was realistic but lacked authentic data (Tor). For instance, in Activity 2, Tor's original task was situated in a real-life context; however, there were no authentic pictures or data (see **Figure 4**). His technique for making his task authentic included suggesting the use of real-life brands and providing real prices for the problem situation.

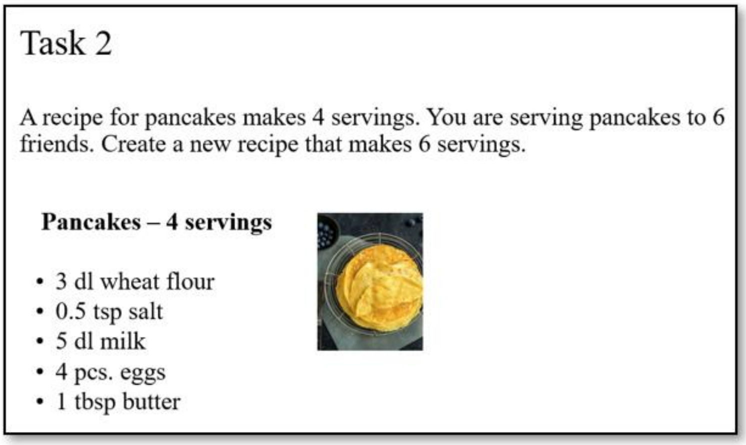

Figure 5. Nora's original task before Activity 2 (This figure displays the translated version of the participant's task, originally in Norwegian. It is translated by the authors)

Two of the original tasks were authentic (Elin and Nora), no changes were needed for these tasks. After Activity 2, five of the tasks were found to be authentic except for Frida's task, which lacked authentic data despite having use value. Moreover, all tasks, whether original or revised, were found to be culturally responsive within the Norwegian context, as they involved situations linked to Norwegian culture.

In terms of structure, there were five tasks involving ratios (Olivia, Nora, Elin, Tor, and Frida), among which two also involved proportions (Nora and Tor). One exception was Lars's task, which required using algebraic equations rather than ratios and proportions.

In terms of functioning, three of the original tasks involved a part-to-part ratio (Olivia, Tor, and Frida), and four involved a part-to-whole ratio (Nora, Elin, Tor, and Frida). The original tasks containing proportions were characterized as missing-value problems (Nora), in which a missing quantity could be determined by forming a ratio with the other known quantities, and as numerical-comparison problems (Tor), in which two ratios were to be compared. For instance, Nora brought the task in **Figure 5** to Activity 2.

In terms of utility, participants' original and revised tasks demonstrated intra-mathematical utility in the domains of algebra (Lars), probability (Elin), and numbers (Nora, Olivia, Tor, and Frida). Participants' original and revised tasks contained the same extra mathematical utility. They provided different examples, such as grocery shopping (Lars), recycling paper (Olivia), a probability game (Elin), and making pancakes (Nora).

DISCUSSION

The importance of task design has been acknowledged in mathematics education (Watson & Ohtani, 2015), and it is also crucial for teachers to be able to select and adapt tasks, especially when teaching heterogeneous student groups (Bardy et al., 2021, 2024). Therefore, addressing the principles of IED is essential, as these principles have been regarded as important in Nordic countries (Frønes et al., 2020) and internationally, as reflected in the broader mathematics education literature, including in Brazil and Germany (Kollosche et al., 2019).

Incorporating IED into task design is crucial for creating inclusive learning environments for all students (Canogullari & Radmehr, 2026). Principles such as LTHCWW and meaningful context can be applied in task design to promote inclusion and equity among learners (Canogullari & Radmehr, 2026). Tasks designed with the principles of LTHCWW have been suggested for their ability to make learning both accessible and challenging for learners (Boaler, 2015; Gadanidis, 2012; Gadanidis & Hudges, 2011). Empirical work has also shown that low-threshold and high-ceiling tasks are effective for mathematics learning for diverse student groups (Falculan & Aberin, 2024). Bardy et al. (2021) also considered openness and task difficulty important features for adapting tasks to students with different achievement levels. In addition, the importance of realistic contexts in tasks has been acknowledged as useful for helping learners see the relevance of mathematics in the real-world (van den Heuvel-Panhuizen, 2003).

The current study aimed to explore the task design practices of pre-service and in-service teachers in addressing IED principles through the lens of ATD. The workshop offered participants an opportunity to reflect critically and engage deeply with the IED principles by providing a set of example techniques they could use in their task design practice. This led them to reflect on the features of their tasks and, if necessary, adjust them to align with the principles given using the available techniques. These results confirm previous studies that have also shown the benefits of reflecting on task design practice for both pre-service and in-service teachers through participation in a professional development workshop, especially in improving their didactical knowledge (e.g., Gomes et al., 2022).

One observation in the first activity regarding PPRM was that in-service teachers brought low-threshold tasks, while pre-service teachers' tasks were characterized as high-ceiling. A possible explanation for teachers' tendency to bring only low-threshold tasks is that they might have aimed to include all students in classroom activities by starting with an entry-level task that everyone can access. Nevertheless, it remains essential for teachers to hold high expectations for their pupils, as this has been shown to advance their efforts to foster equitable educational opportunities (e.g., Yılmaz et al., 2021). Productive struggle can also be an effective way to encourage students to recognize that challenge is a natural part of learning mathematics and can be an important consideration in teachers' task design practice (Warshauer, 2015). Therefore, these findings suggest that teachers may intentionally design tasks that incorporate productive struggle to introduce meaningful challenges. Unlike in-service teachers, pre-service teachers' tendency to choose high-threshold tasks may be explained by their training, as they may be taught to design cognitively more challenging tasks in teacher education courses. Even though pre-service teachers had opportunities during the workshop to reflect on adjusting the difficulty of their tasks, they still faced challenges in making their tasks more accessible, indicating a potential area for improvement in their practice.

In the second activity, the workshop supported participants in shifting the context of their tasks from dressed-up to realistic, from dressed-up to authentic, and, for some, from realistic to authentic. The first shift was achieved by adding justification to dressed-up tasks, making their contexts more realistic without necessarily including authentic data or images. The second and third shifts occurred when participants either provided or suggested including authentic images or data to enhance the authenticity of the context. Although previous research suggested that pre-service teachers encounter difficulties in selecting meaningful contexts (e.g., Sujadi et al., 2025), the pre-service teachers in our study demonstrated the ability to adapt their tasks to enhance realism and authenticity during the workshop, thereby underscoring the advantages of such activities for their professional growth. The results also showed that all participants brought culturally relevant tasks for this activity. This may indicate that cultural relevance is a high priority for both pre-service and in-service teachers in mathematics teaching in Norway. This result is consistent with prior research indicating that many pre-service elementary mathematics teachers recognized the importance of acknowledging students' diversity in their cultures and backgrounds (e.g., Jackson & Jong, 2017).

Another important result for both activities and both groups was that, in terms of the ERM's structure, there were both ratio and proportion tasks both before and after the workshop, meaning that participants did not change the structure of their tasks. This was expected because they were asked to bring tasks related to ratios and proportions. However, regarding the functioning and utility of ERM, several results stood out. For instance, the ratio as a rate was never used by any participant in either activity. This might have been explained because presenting the rate would require a context in which the ratio was interpreted as a rate; however, in the first workshop activity, they were asked to bring bare tasks. Although they were not told to provide context for their bare tasks in the first activity, one pre-service and one in-service teacher still did. On the other hand, for the second activity, they were asked to bring in contextual tasks, and both the functioning and utility of some participants' tasks changed after the workshop. This suggests that when context is involved, there may be greater flexibility in addressing the functioning and utility of mathematical knowledge. The functioning can be addressed more explicitly by introducing a broader range of concepts relevant to learning ratios and proportions, such as fractions and percentages (ratio as part-whole), division (ratio as a quotient), and rates (ratio as a rate). This might also increase its intra- and extra-utility, depending on the context integrated into the task.

The study also has limitations regarding the transferability of its results due to methodological choices. First, it is important to note that the results should be interpreted within the context of the Norwegian education system, since the participants were working in Norwegian institutions. Furthermore, due to the small sample size, the findings cannot be generalized to all Norwegian pre-service and in-service teachers, as the results may reflect individual differences in their practices rather than a culturally influenced approach to task design; thus, caution is warranted when interpreting them. Furthermore, while we provide implications that may be transferable to classroom settings and at the disciplinary level, we do not claim to offer solutions for addressing IED at the school, institutional, or country level, as each of these institutions has different norms, regulations, and rules that may affect how our practical suggestions are carried out by teachers. Furthermore, our ERM was designed for primary and lower secondary levels. Future studies could extend its applicability to upper secondary levels by incorporating additional concepts into its structure, such as linearity, similarity, and rate of change. Additionally, we worked with a small sample size in this study, which was primarily selected at convenience. Future research could replicate the workshop with pre-service and in-service teachers across different levels, contexts, and cultures, to enrich and substantiate these results with broader perspectives.

CONCLUSION

To conclude, in-service mathematics teachers who aim to design tasks that foster inclusion and equity may benefit from reflecting not only on the didactical aspects of their tasks (such as the IED principles) but also on the structure, functioning, and utility of the mathematical knowledge within those tasks. The PPRM we developed can serve as a reflective tool for both research and teaching, when combined with teachers' own techniques and technologies, to support the integration of IED principles into task design across various mathematical concepts. Future studies can enrich our PPRM with additional principles that may be valuable for fostering inclusion and equity in classrooms. For instance, the use of digital technology has been suggested as a means of creating low-threshold, high-ceiling, and wide-wall tasks for grade 3-grade 5 in the context of area measurement (Stehr et al., 2018). Technology-integrated tasks that address the LTHCWW and meaningful context principles might be valuable for future investigation and could be included in subsequent PPRMs.

In addition, mathematics educators may use the PPRM and ERM in teacher education courses for pre-service teachers as well as in professional development workshops for in-service teachers. Given the provisional nature of reference models (Gascón & Nicolás, 2022), the PPRM and ERM are modifiable, allowing researchers to adapt them to their own cases.

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