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Research Article

Mathematics in vocational education: A comparative study between Portugal and Spain

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ABSTRACT

Vocational education is an international priority, as it contributes to countries' economic growth, quality education, and social development. This article aims to compare vocational education's 'prescribed' and 'enacted' curricula (curriculum implementation guidance documents, textbooks, classroom resources, materials, and research articles) in Portugal and Spain. Adopting a qualitative and interpretative methodology, the study focuses on the mathematics discipline of vocational courses at the secondary level organization within compulsory education. The comparison of curricula in Portugal and Spain was sought, using the conceptual framework developed in 1971 by Heinrich Roth, which was applied in 2014 by Pilz et al. (2014) and later in 2022 by Rodrigues and Pimenta (2022). The conclusions emphasize the diversification of mathematical themes in both curricula and the focus on developing transversal skills to prepare active and engaged citizens, particularly by analyzing the role of mathematics in vocational education. Based on the findings, the curriculum should prioritize the diversification of mathematical topics, provide clear guidelines for mathematics teaching and learning, promote the development of transversal skills to support lifelong learning, and create resources and training to enable teachers.

Keywords: curriculum development, teaching and learning mathematics, curricular innovation, vocational education

INTRODUCTION

Vocational training can provide young people with the skills necessary for work and lifelong self-learning, contributing to adaptation to different professional contexts (Rodrigues, 2018). It can also improve the capacity to respond to social and technological changes by meeting the needs of companies and communities, promoting inclusive and sustainable economic growth, and supporting transitions to green economies and environmental sustainability (UNESCO, 2016).

One of the assumptions of the 2030 agenda for Portugal is to substantially increase the number of young people and adults with relevant qualifications. Considering that training and professional education have repercussions on the labor market and economic development policies (SDG8), the promotion of sustainable development (SDG12), and quality education (SDG4), it is important that vocational courses are aligned with the needs and dynamics of the labor market. Vocational education becomes relevant regarding the quality of response; students can return to society by providing them with skills, knowledge, and qualities that promote their self-confidence and lifelong learning skills (European Commission, 2021; Rodrigues, 2018). Knowledge about the subjects is essential to promote the acquisition of skills (Vinão, 2007), as these are born, evolve and incorporate learning and knowledge (Almeida, 2021; Santiago, 2021), change their name and content (Matos et al., 2019) and promote the development of the ability to be a citizen and a professional in the 21st century (Rodrigues, 2015).

Mathematics and science skills, measured by the TIMSS or PISA assessments, track international differences in economic growth and are a good metric for assessing the potential of a country's labor force (Hanushek, 2013). In this sense, we will focus the study on the organization of the mathematics discipline curriculum of vocational courses at the secondary level (compulsory education) in two European countries, Portugal and Spain. This article aims to systematize information about the mathematics curriculum defined for vocational courses and simultaneously verify the contribution of mathematics to the construction of professional profile. We adopted the concept of Gimeno (2000), for whom the curriculum is broader than that written in legislative documents, designates a complex cultural and social concept, and establishes a bridge between society and school (Rodrigues, 2015, 2018).

Cai et al. (2016) stress the importance of international comparative studies in mathematics education "because they provide a large body of knowledge showing how students do mathematics in the context of the world's varied educational institutions. They also examine the cultural and educational factors influencing mathematics learning" (p. 1). Khakpour (2012) argues that comparative studies comparing the content and process of successful education systems can be helpful to design and implement educational changes. About comparative technical and vocational education and training (TVET) research, Clarke et al. (2020) said that it has been transformed and enriched by the European dimension, moving away from simply comparing different typologies on a country-to-country basis.

In this article, analyzing the self-competence category allows us to reflect on the importance of transversal skills for preparing active and intervening citizens, particularly analyzing the role of mathematics discipline in vocational education. To contribute to a reflection on the organization of the mathematics curriculum, a comparative analysis was made between the curriculum of two countries (Portugal and Spain) to see transversal dimensions of learning that are important for the training and integration of young people in the business market. In this context, we sought to answer the following research questions:

1. What are the convergences and divergences in vocational secondary education systems?
2. In different curricula, what are the most focused mathematics topics?
3. Is there social relevance between mathematics taught and professional profile related to the training area?

LITERATURE REVIEW

To UNESCO (2016), one of the goals of education 2030 is to develop quality through TVET. TVET is expected to address "the multiple demands of an economic, social and environmental nature by helping youth and adults develop the skills they need for employment, decent work and entrepreneurship, promoting equitable, inclusive and sustainable economic growth, and supporting transitions to green economies and environmental sustainability" (UNESCO, 2016, p. 4). To achieve this, one of the most important challenges is to structure and organize education systems. To prepare students for challenges in the future, they must be equipped with the skills, knowledge, and qualities necessary to feel confident and have lifelong learning skills (European Commission, 2021; Rodrigues, 2018).

Despite the importance of acquiring multidisciplinary skills (social, communication, thinking skills, etc.), knowledge about the disciplines will continue to be important as it will support background acquaintances, such as knowing how a mathematician, historian, or scientist thinks (OCDE, 2018). Also, disciplinary knowledge will continue to be important as it will support the knowledge to develop the capacity to be a citizen in the 21st century (Rodrigues, 2015). According to Vinão (2007), school subjects are not static. They are born and evolve, incorporating learning and knowledge of educational agents (Almeida, 2021; Santiago, 2021). They change or disappear, move away and unite, and repel and absorb each other (Matos et al., 2019).

To reflect on the importance of multidimensional competencies, occupational capacities, knowledge, general and civic education, permeability with other educational systems, and connection with the labor market, and, in particular, to analyze the role of mathematics in TVET, we will consider the importance of comparative studies and look at how educational policies are reflected in vocational education curricula and what has been done to train qualified professionals.

Magalhães (2011) compared the self-concepts of competence and self-learning of scientific-humanistic and technical and vocational education students. This work enhances the importance of analyzing the competencies associated with these constructs as a predictor of school achievement and social integration. It shows significant differences in favor of students who attend scientific-humanistic courses and perceive themselves as more competent in the cognitive and social domains. Magalhães (2011) also mentions the importance of developing studies on these constructs in technical and vocational education contexts.

In turn, Gonçalves et al. (2018) comparative study explores the differences between the 'prescribed' and the 'enacted' curriculum, arguing that although it is possible to identify distinct curricula in different countries, vocational education is taken seriously across the board, despite a few differentiating factors such as the resources availability, culture, social and economic differences.

A comparative research project on prevocational education in middle schools in seven European countries was presented by Pilz et al. (2014), using the conceptual model developed by Heinrich Roth in 1971. To the authors, this model is well-suited to international comparisons and is also in line with the components of the European qualifications framework. Drawing on Pilz et al. (2014), Rodrigues and Pimenta (2022) compared the prescribed mathematics curriculum for professional secondary education and the accompanying documents that guide its implementation, including suggested resources and tasks for classroom use in Portugal, Spain, and Singapore. In doing so, they identified and highlighted the divergences and convergences in three areas of competencies: knowledge-based competencies, self-competence, and social skills. This analysis aimed to understand how curriculum design in different countries could facilitate the acquisition of multidimensional competencies. Additionally, they examined how the curriculum ensures the development of social skills.

METHODOLOGICAL OPTIONS AND PROCEDURES

A relevant action is to systematize information on the mathematics curriculum in TVET and verify the contribution of mathematics to constructing professional profile and developing essential competencies for citizenship. The two countries of the Iberian Peninsula were selected for this comparative study, although the employability rate of graduates from professional courses in Spain is lower than in Portugal (European Commission, 2021). Mathematics curricula and pedagogical guidance documents approved in Portugal and Spain were collected.

This study follows an interpretative qualitative methodology supported by documentary curriculum analysis and other documents supporting curricular implementation in the countries under study. It used the conceptual model developed by Heinrich Roth in 1971, applied by Pilz et al. (2014) and Rodrigues and Pimenta (2022), which aims to analyze how and to what extent educational components are incorporated in the countries covered by the study. This model is suitable for international comparisons, and despite the date it was created, it is aligned with the European qualifications framework, which justifies its use in current research. The model considers three categories of analysis: knowledge-based competencies, self-concept competencies, and social competencies. Knowledge-based competencies are analyzed into topics and themes, program structure, and assessment. The self-concept competencies focus on purpose and action, which is visible in the methodologies associated with the curriculum or in complementary documents and contributes to entrepreneurship and building skills for lifelong learning. Social competencies focus on analyzing the potential of the curriculum to develop personal and interpersonal skills.

The convergences and divergences in teaching mathematics in vocational education and the subject's contribution to developing transversal skills for integration into active life or continuing studies were studied. The content and organization of mathematics, objectives, methodological guidelines, interrelationship, articulation of mathematics with other subjects, teaching and learning methodologies, and the skills, attitudes, and values students need to prosper were integrated. We emphasize that the present study was

Table 1. The model applied by Pilz et al. (2017) and by Rodrigues and Pimenta (2022)

	Areas	Sub-areas
Curricular analysis mathematics	Knowledge-based skills	Themes Program structure Assessment
	Skills based on self-concept	Purpose Action
	Social skills	Personal Interpersonal

conducted independently of the comparative analysis involving Portugal, Singapore, and France. Therefore, no comparisons were made between Portugal, Spain, and the countries examined in the previous study. Nonetheless, it ensured the application of the same analytical criteria in both studies, maintaining consistency in defining categories and subcategories of analysis, as presented in **Table 1** of the data analysis section.

EDUCATIONAL SYSTEMS IN DIFFERENT COUNTRIES

The international standard classification of education (ISCED) constitutes the reference classification that makes it possible to organize education programs and corresponding qualifications by educational levels and areas of study. This results from an international agreement and was formally adopted at the general conference of UNESCO member states. Internationally,

> vocational education is defined as education programs that are designed for learners to acquire knowledge, skills and competencies specific to a particular occupation, trade, or class of occupations or trades. Such programs may have work-based components (e.g., apprenticeships and dual-system education programs). Successful completion of such programs leads to labor market-relevant, vocational qualifications acknowledged as occupationally-oriented by the relevant national authorities and/or the labor market (UNESCO, 2011, p. 14).

Within the scope of this article, we will classify vocational education at the secondary level in ISCED. We will not deal with the specificities of vocational education at the post-secondary level. In both countries, vocational education plays a differentiating role, noting that vocational training can provide young people with the skills necessary for the world of work and for lifelong self-learning, contributing to adaptation to different professional contexts (Rodrigues, 2018). It can also improve the capacity to respond to social and technological changes by meeting the needs of companies and communities, promoting inclusive and sustainable economic growth, and supporting transitions to green economies and environmental sustainability (UNESCO, 2016).

Portugal

Compulsory education lasts 12 years and begins in the first cycle (4 years), continues with the second and third cycles (2 years and 3 years, respectively), and concludes with secondary education (3 years). Vocational education lasts 3 years and is one possible path to completing secondary education. It allows double certification: academic (12th year) and professional (according to the desired qualification). Upon completion, students reach the required age and obtain the qualification that enables their entry into the labor market with a European level 4 professional certification. Thus, completing these courses enables students to obtain a secondary education completion certificate, which allows them to pursue further studies. To do so, they must either sit for the national access exam or apply for a higher professional technical course. This is a higher education cycle that does not confer an academic degree but, upon successful completion, grants a diploma as a higher professional technician (UE level 5) and allows them to apply for a bachelor's degree program.

Vocational secondary education courses are the responsibility of the national agency for qualification and professional education (ANQEP) and are closely linked to the professional world. They target students between 15 and 20 who have completed basic education, specifically the ninth year of schooling (Rodrigues, 2015).

ANQEP defines the professional profile associated with the 394 qualifications, with programs available in the national qualifications catalog. The training plan for these courses is organized into modules and includes three training components: sociocultural, scientific, and technical. The latter must include 600 and 840 hours of training in a work context, as defined by the school. To complete the courses, it is necessary to present a final project: professional aptitude project (PAP). Mathematics is included in the scientific component of the courses and has a workload of 100, 200, or 300 hours (Rodrigues, 2015). The program for this subject (essential learnings in Portugal), aligned to *students' profile by the end of compulsory education*, was approved in January 2023 (Carvalho e Silva et al., 2023; Martins et al., 2017). In 2023/24, it began to be implemented in some pilot classes and will be generalized to all secondary school classes in the 2024/25 academic year.

Spain

Basic and compulsory education lasts ten years, with primary education lasting six years and the second cycle four years. At the end of the second cycle, the student reaches the legal age in Spain to enter the job market. Every child, regardless of nationality, has access to public education. Enrolling in a public school is not the same in different autonomous communities.

Vocational education is one possible route to completing secondary education. Attendance at a basic level course lasts two or three years (*grado básico*), and it is the end of compulsory education in Spain, which attributes the student to a European level 3A qualification. After compulsory schooling, students must complete a secondary vocational course to access higher education courses (*grado medio–EU level 4*). In Spain, after completing secondary education, students can choose from various pathways, including vocational education. *Grado medio* lasts for two years and provides access to both the job market and higher education, contingent upon successfully completing an entrance exam. Access to university is granted through the university access exam or, for those who have completed vocational training cycles, the access exam for vocational training students. However, students who have completed the higher vocational training cycle (*grado superior*) can access the university directly after finishing the cycle, potentially being exempt from certain exam courses.

In conclusion, compulsory education lasts 12 years in Portugal and 10 years in Spain. However, in Spain, an additional 2 years of vocational training, following compulsory education (*grado medio*), is available for those aiming to obtain an EU level 4 qualification. Similar to Portugal, in Spain, basic training cycles are targeted at students whose preferences and expectations align with professional realities, who intend to integrate into the labor market, and who have completed 15 years of age or will do so until December of the first year in the course, as well as having completed the third course of compulsory education (Royal Decree, 2023).

Basic technician titles have official character and validity throughout the national territory and do not constitute a regulation of professional exercise. This basic degree training cycle consists of 3 different areas, projects, and tutoring:

(1) communication and social sciences area,
(2) scope of applied sciences, which will include, in an integrated manner, applied mathematics and applied sciences,
(3) professional scope, which will include the professional modules linked to level 1 competency standards from the national standard catalog of professional skills, which will include the personal Itinerary module for its employability,
(4) intermodular collaborative learning project, and
(5) tutoring (Royal Decree, 2023, 2024).

The curriculum applied sciences is fixed in annex V of Royal Decree (2022), of March 29, aligned to *students' profile by the end of compulsory education*, and this discipline has a duration of 115 hours (60 and 55, respectively on applied science I and applied science II) (Royal Decree, 2022). Training in a work context has a workload of 20% of the training cycle (Royal Decree, 2023). Implementing the first course of all training cycles will be completed in the 2024/25 school year (Royal Decree, 2024).

Table 2. The organization of themes in the curricula of 2 countries under analysis

Country	Themes
Portugal	The defined themes depend on the chosen course and are defined by modules, up to 6 mandatory modules according to the number of hours of mathematics course, and 18 optional modules. Geometry (includes trigonometry), statistics, probabilities, real functions of a real variable; discrete mathematics, mathematics for citizenship and mathematics applications are mandatory. Transversally, there is an interest in promoting computational thinking, implementing mathematical modelling activities and application to the real context and an approach to the history of mathematics.
Spain	Mathematics is integrated into the professional modules of applied sciences. Applied mathematics and applied science subjects are integrated in the same scope. The knowledge of applied mathematics is grouped into the same senses in which they are articulates the subject of mathematics in compulsory secondary education: the acquisition of number sense is characterized by the application of knowledge about numeracy and calculation in different contexts, especially professional ones; the sense of measurement centers on understanding and comparing attributes of objects; the spatial sense approaches it understanding the geometric aspects of our world; the algebraic sense provides the language in which mathematics and science are communicated; lastly, the stochastic sense comprises the analysis and interpretation of data and comprehension of random phenomena to support decision-making at work level and, in general, in a world full of uncertainty.

DATA ANALYSIS

A qualitative methodology of an interpretative nature was adopted, and mathematics programs and some approved pedagogical guidance documents were analyzed. Mathematics themes and their relation with professional profile, curriculum structure, teaching approaches, assessment, and the social relevance underlying the adopted curriculum were investigated to identify convergences and divergences in secondary vocational education systems. To analyze and organize the data collected, a table was created based on the conceptual model developed by Heinrich Roth and applied by Pilz et al. (2014) and by Rodrigues and Pimenta (2022).

Knowledge-Based Skills

Regarding knowledge-based skills, the subcategories' themes, program structure and assessment were analyzed to understand how each curriculum seeks to ensure essential mathematics skills. The first subcategory explains the mathematics topics adopted. The structure subcategory corresponds to the constructed learning design and reflects, among other structural aspects, how students can acquire the expected mathematics skills. This subcategory will identify the specific organizational and structural aspects of the adopted curricula. In the assessment subcategory, we seek to understand how each defined curriculum intends to measure the acquisition of mathematics skills (Pilz et al., 2014; Rodrigues & Pimenta, 2022).

Table 2 reflects the themes studied throughout secondary vocational education in Portugal (Carvalho e Silva et al., 2023) and Spain (Royal Decree, 2022).

The choice of topics to teach is very different in both countries. The Spanish curriculum focuses on integrating mathematics in applied sciences and developing the sense of numbers, measure, geometric, algebraic, and statistics. All topics covered directly relate to learning another science, such as physics, chemistry, biology, or geology. The Spanish curriculum emphasizes the study of numbers, linking them to problems addressed within applied sciences. In contrast, the Portuguese curriculum integrates this learning and training in earlier school years, considering this knowledge on a basic level of education. The relationship with other subjects is, in Portugal, directly linked to the title of the module to be taught, such as biomathematics, mathematical models for citizenship, and fiscal mathematics, among others; in Spain, on the other hand, it does not exist a mathematics discipline and this subject it is integrated with other sciences in applied sciences subject. In summary, in Portugal, the teaching of mathematics follows a practical and applied approach, ensuring the acquisition of knowledge on the topics defined for the subject while maintaining a strong connection with the respective areas of vocational training. Despite this, there is also significant integration with other fields of knowledge. In Spain, however, mathematics does not maintain the same level of independence as it is integrated with various scientific disciplines; consequently, mathematical topics emerge within the broader context of learning other scientific subjects.

Regarding their organization, the structure of the different programs is divergent. In this article, we systematize the structure in **Table 3** based on Carvalho e Silva et al. (2023) and Royal Decree (2022).

Table 3. Program structure in the curricula of 2 countries under analysis

Country	Mathematics program structure
Portugal	The mathematics program is divided into 4 areas: (1) themes and topics, (2) learning objectives, knowledge, skills, and attitudes that the student must reveal, (3) strategic teaching actions by the teacher, where the role of the teacher is clarified, methodological indications are given and proposed activities to be developed with students, and (4) skills in the *students' profile by the end of compulsory schooling*. The structure of the different modules includes clarifying notes, namely on activities that promote the development of computational thinking, with resources for example, proposals for possible further developments or alternative approaches, bibliographical references, and resources to support the teacher's work. In line with the *students' profile by the end of compulsory schooling*, it is intentional to develop transversal skills, such as solving real problems, applying appropriate mathematician procedures, and interpreting the results in several real contexts. Formulation of hypotheses, testing conjectures, deduction, generalization, and abstraction, are part of the curriculum. It intends to develop students' awareness of the growing relevance of the role of mathematics in current society on issues such as electoral processes or financial models. It valorized statistical literacy and computational thinking, in the analysis of real-life situations, the identification of mathematical models that allow their interpretation and resolution, the selection of strategies to solve problems, the formulation of hypotheses and prediction of results contemplating mathematical modelling, experimental work and the study of situations in a real context suited to the professional profile of each course. Priority should be given to solving non-routine problems, connections between mathematician themes, and applications of mathematics in other disciplines, with the resource of technology.
Spain	The program is organized into (1) specific skills and (2) assessment criteria. In the field of applied mathematics, there are 6 fundamental knowledge areas, known as basic knowledge, which include numerical sense, measurement sense, spatial sense, algebraic sense, and statistical sense. In addition to these, there are 2 additional knowledge–basic scientific skills and socio-affective sense–which should be explicitly developed throughout the curriculum. Furthermore, there are 8 specific competencies defined, which align with the *students' profile by the end of compulsory schooling*. These competencies intersect with STEM, computational thinking, and methodologies inherent to technical and scientific work, emphasizing the importance of collaborative work and promoting skills that address personal and social well-being and raise awareness of environmental sustainability issues. Furthermore, it emphasizes the importance of teaching math with real-life applications. Additionally, it outlines assessment criteria for the 6 competencies defined.

The organization and structure of the curricula in the two countries differ. Portugal emphasizes teaching methodologies and pedagogical strategies, while Spain, after explaining the themes, provides more detailed assessment criteria. The Spanish curriculum explicitly references the development of socio-affective competencies, whereas the Portuguese curriculum implicitly addresses these aspects, linking them to the *students' profile by the end of compulsory schooling*. The structure presented in Portugal appears more organized, facilitating a clearer interpretation.

Both programs aim to develop the transversal skills outlined in the *students' profile by the end of compulsory schooling*, a guiding document in both countries. However, the Spanish curriculum offers general guidelines for skill development and assessment criteria. In contrast, the Portuguese curriculum includes teaching actions for educators and a list of references. Both programs prioritize an applied approach to mathematics, emphasizing the resolution of real-world problems connected to other fields of knowledge. To achieve this, they foster the development of computational thinking, mathematical modelling, and science, technology, engineering, and mathematics (STEM) methodologies.

In the Portuguese curriculum, clarifying notes, examples of activities to develop computational thinking, proposals of possible deepening or alternative approaches, bibliographical references, and resources to support the teacher's work add value because they direct the teachers' work.

In the Spanish curriculum, applied mathematics is incorporated into the discipline of applied sciences, aiming to develop interdisciplinary relationships with physics, chemistry, biology, or geology.

Both programs explain the intentions and guidelines for teaching mathematics and provide instructions on assessment, which are systematized in **Table 4**, based on Carvalho e Silva et al. (2023) and Royal Decree (2022).

The curriculum approach to assessment is very different in both countries. In Spain, the evaluation criteria for each competence are presented similarly to what appears in the Portuguese curriculum in the column referring to strategic actions for teacher teaching. Formative assessment is presented globally and is not specific to the subject of applied sciences, such as the qualitative scale.

Table 4. The assessment of the discipline in the curricula of 2 countries under analysis

Country	Assessment
Portugal	The program values the formative nature of the assessment, which must involve learning activities carried out in class and the application of diverse instruments. Some tasks can be designed to constitute a summative assessment, to classify learning. To continue higher-level studies, the assessment method allows you to take an exam, which covers the 6 mandatory modules. The assessment at the end of each module is quantitative, on a scale from 1 to 20.
Spain	Assessment criteria relating to the skills to be developed in the subject are presented and the basic knowledge to be acquired is listed. There is a general reference to the type of assessment for compulsory education, it is stated that it will be continuous, formative, and integrative. But there is no reference to the instruments. Assessment criteria are presented, related to specific skills to be developed. Understanding, analysis, and interpretation are valued and, in some cases, the use of technology is explicitly requested. The assessment is qualitative, and the results are expressed in the terms "insufficient (IN)", for negative qualifications; "enough (SU)", "good (BI)", "notable (NT)", or "spare (SB)" for positive ratings.

In Portugal, clear indications are given about the importance of prioritizing formative assessment, and instruments for collecting information are suggested for evaluating students. In this country, given the course structure by modules, the assessment takes place at the end of each module, and there is no provision for a global quantitative assessment at the end of each year. By analyzing the criteria of the Spanish curriculum, it appears that each competency can be assessed in isolation without being interrelated with the others.

Skills-Based on Self-Concept

Through the analysis of the skills based on the self-concept category, we seek to illustrate how the curriculum design of different countries can contribute to the acquisition of competence for lifelong learning based on the key competencies defined by the European Union (European Commission, 2019). This category was divided into two subcategories: *purpose*, which refers to the intentionality of teaching mathematics, and *action*, which contains guidelines transmitted through the prescribed curriculum (Gimeno, 2020).

Table 5 presents the purpose of the curriculum in both countries based on Carvalho e Silva et al. (2023), Portaria (2018), and Royal Decree (2022, 2024).

Table 5. Purpose of the curriculum in the curricula of 2 countries under analysis

Country	Purpose
Portugal	The training of mathematically competent individuals is a fundamental purpose of the mathematics curriculum for secondary education. The curriculum aims to provide young people with transversal skills that enable them to learn throughout their lives and support professional development in a rapidly evolving labor market. It also enshrines the purpose of preparing students to formulate judgments and make informed decisions, helping them to become reflective, committed, and participatory citizens. It also aims to help young people appreciate the role of mathematics in the world and its character as a science in evolution and permanent renewal, appreciating its aesthetic dimension alongside its historical legacy. This courses intended to promote the develop of personal or professional competencies among students, so integrated into the curricular plan, students have to develop an PAP, which consists of the presentation and defense, before a jury, of a project embodied in a product, material or intellectual, of an intervention, or a performance, depending on the nature of the courses, as well as the respective completion and critical appraisal report, demonstrating the knowledge, skills, attitudes and professional skills acquired throughout the student's training path, in all training components, with a special focus on the areas of competence registered in the *students' profile by the end of compulsory schooling* and in the professional profile associated with the respective qualification. PAP is part of the external assessment.
Spain	The discipline of applied sciences intends to contribute to the integral formation of the student that requires the understanding of concepts and scientific procedures that allow them to develop personally and professionally; involve students in questions related to science, reflect on them; and develop in a world that continues to develop scientific, technological, economic, and social, to be able to integrate into democracy society as committed citizens. In addition to the development of mathematical skills, the curriculum emphasizes the development of affective sense, which integrates knowledge, skills, and attitudes to understand and manage emotions, establish and achieve goals, and increase the ability to make responsible and informed decisions, which are aimed at improving the performance of the student in mathematics, to the reduction of negative attitudes there were, to the promoting active learning and eradicating related preconceived ideas with the genre or the myth of indispensable innate talent. Integrated into the curricular plan, students have a module, with the duration of 25 hours, called "intermodular collaborative learning

Table 5 (Continued).

Country	Purpose
	project", which is divided into four phases: (1) search for information on the internet about companies whose activity is related to the area of the courses and draw up a map of the services they offer; (2) select a service provided by the company and relate it to the SDGs and recipients at a global level, (3) prepare a proposal for a "spin off" company, indicating the differentiating aspects in relation to the previously selected company, and (4) list each company unit with the prevention of professional risks.

Table 6. Curriculum action in the curricula of 2 countries under analysis

Country	Action
Portugal	The curriculum presents, for each domain, the teacher's strategic teaching action, indicating methodologies to be applied in class and suggestions for incorporating computational thinking, in addition to the historical context of mathematics and its application to the real world. Problem-solving, the use of varied contexts, mathematical modelling, and the use of digital technologies, among other suggestions, are transmitted as a working methodology in the classroom. Some specific suggestions are given for teaching certain topics, but also for promoting transversal skills, essential for the integral development of students, such as abstraction, rigor, systematic verification, and control practices, among others. "Possible further developments" are also suggested, to be developed in a classroom context, with the presentation of different application examples.
Spain	The curriculum consists of two main areas of study that must be developed continuously: "Basic scientific skills" (mathematics, biology, physics, geology, and chemistry) focusing on the strategies and critical thinking essential to the sciences, and "socio-emotional understanding", which aims to help students acquire and apply techniques to understand and manage emotions, set and achieve goals, and demonstrate empathy, solidarity, respect for minorities, and gender equality in scientific and professional activities. It is emphasized that the presentation of knowledge does not imply any chronological order, as the curriculum has been designed as an integrated whole. Specific competencies are defined, and to develop them, the use of methodologies inherent to science and digital technologies is proposed, approached with an interdisciplinary, co-educational focus connected to the reality of the students. The aim is for learning to acquire meaningful significance through planning learning situations, preferably linked to personal, social, and professional contexts. It aims to involve students in the challenges and objectives of the modern world and the SDGs, helping them integrate professionally and actively participate in a democratic and diverse society.

The similarities between the countries regarding the purpose of the curriculum are clear. The subject of mathematics (Portugal) or applied sciences (Spain) aims to contribute to the training of active and informed citizens with skills to enter the job market or pursue higher education studies. All documents highlight transversal adaptability skills that allow integrated citizen action and lifelong learning, with adaptability to different contexts.

In both countries, there is a clear objective of encouraging the development of skills in the *students' profile by the end of compulsory schooling*; however, in the Spanish curriculum, there is a notable focus on developing socio-affective skills about others and mathematics itself. In both countries, students develop a project. In Portugal, this is not covered modularly, and students develop an interdisciplinary project related to the learning and skills developed in the professional course, presented in front of a jury and which hold relevance for internal and external assessment. In Spain, this project is just another module that aims to integrate entrepreneurship and the proposal to create a company, which is integrated into a 25-hour module, relevant only for internal assessment. In both countries, students achieve, after completion, two certificates, one related to professional competencies, aligned with the European qualifications framework and an academic diploma equivalent to the conclusion of 10th grade in Spain and 12th grade in Portugal.

Table 6 presents the guidelines transmitted through the prescribed curriculum, which is based on Carvalho e Silva et al. (2023) and Royal Decree (2022). The Portuguese curriculum guides teachers' actions more precisely, giving more concrete methodological indications, strategies and suggestions. The Spanish curriculum does not present such precise indications, but the need to develop fundamental scientific and socio-affective skills using digital technologies and collaborative work is evident.

The curricula of both countries under study highlight the importance of preparing for real life and emphasize the mathematics curriculum's focus on problem-solving skills. Although each country has opted for a different organization of curricular documents, in addition to the content to be taught, the teaching and learning methodologies for the subject are detailed, as are the proposals for evaluation.

Table 7. Personal skills in the curricula of 2 countries under analysis

Country	Personal skills
Portugal	Pedagogical suggestions are provided to teach "mathematics for citizenship" and "applications of mathematics," aiming to form creative citizens capable of making decisions and affecting change. By solving problems and engaging in project development, the goal is to stimulate the development of reasoning, creativity, autonomy, rigor, organization, confidence, determination, and communication skills, which are essential for the holistic development of the individual. It is suggested to provide continuous feedback to students during activities so that they can engage in the learning process, reflect on their work, and correct their mistakes. The curriculum aims to develop the skills of the *students' profile by the end of compulsory schooling*, such as languages and texts; personal development and autonomy; information and communication; well-being; health and the environment; reasoning, and problem-solving; aesthetic and artistic sensitivity; thinking critical and creative thinking; scientific, technical and technological knowledge; interpersonal relationships and awareness and control of the body.
Spain	The specific development of socio-affective skills is a specific focus of the program, at the same level as the development of mathematical skills. The curriculum contributes to the development of skills for permanent learning throughout with an articulated teaching of mathematics with biology, chemistry, physics, and geology. The development of socio-affective sense is guided by the acquisition and application of strategies to understand and managing emotions, establishing and achieving goals, feeling and showing empathy, solidarity, respect for minorities and effective equality between men and women in activity scientific and professional. The specific competencies of the scope are linked directly to the descriptors of *students' profile by the end of compulsory schooling*, like competence in linguistic communication; multilingual competence; mathematics competence and competence in science, technology and engineering; digital competence; personal, social and learning to learn competence; citizen competence; entrepreneurial competence; and competence in consciousness and cultural expression.

Social Skills

After reviewing the earlier analysis, it is clear that the mathematics curriculum emphasizes the development of not only disciplinary skills but also personal and interpersonal skills. Following the Pilz et al. (2014) model, we identified these as social skills. These skills are referenced in both countries' curriculums and supplementary documents that explain the principles of their educational systems.

Table 7 presents the skills that both curricula focus on development, based on Carvalho e Silva et al. (2023), Royal Decree (2022), and *students' profile by the end of compulsory schooling* in both countries (Martins et al., 2017; Royal Decree, 2022).

Both curricula are aimed at developing the personal skills of the *students' profile by the end of compulsory schooling*, a curricular document that exists in both countries. Although the designation of skills is different, the focus of skills development in both curricula is similar. However, the language of the Spanish document is more directly linked to skills development STEM. In STEM, mathematical competence allows you to develop and apply perspective and mathematical reasoning to solve different problems in different contexts. Competence in science involves understanding and explanation of the natural and social, using a set of knowledge and methodologies, including observation and experimentation, to ask questions and draw conclusions based on tests to be able to interpret and transform the natural world and the social context. In Portugal, reasoning and problem-solving skills are concerned with finding answers to a new situation, mobilizing reasoning with a view to decision-making, constructing and using strategies, and formulating new questions.

The Spanish curriculum focuses more on developing socio-affective skills, both personally and in the student's relationship with mathematics.

Table 8 aims to show the interpersonal skills that the curriculum focuses on developing in both countries, based on Carvalho e Silva et al. (2023), Royal Decree (2022, 2023), *national strategy for education for citizenship* (Monteiro et al., 2017), *education strategy for global citizenship* 2030 (Ayuda en Acción, 2020).

In Portugal, the curriculum focuses on developing social and civic competencies, emphasizing active learning, teamwork, and the integration of mathematics with other areas, guided by the national strategy for citizenship education. In Spain, the curriculum highlights social competencies aligned with the sustainable development goals (SDGs), including strategies for acquiring digital and interpersonal skills, with a 50-hour module dedicated to employability. Both curricula promote active citizenship and the development of social competencies, but Portugal places greater emphasis on civic education and integration with other subjects,

Table 8. Interpersonal skills in the curricula of 2 countries under analysis

Country	Interpersonal
Portugal	Specific modules, such as citizenship education, involve students in active citizenship projects to raise awareness and foster civic attitudes. Portugal also has a guiding document known as the *national strategy for education for citizenship*, which contributes to the development of these competencies. Other topics provide essential tools for interpreting information, working in teams, considering diverse perspectives, building consensus, addressing challenges, and establishing harmonious relationships with others. For example, it prioritizes active learning through the facilitation of exploratory group work, encouraging participation towards a common good. According to the specificities of the courses, programs stimulate the development of financial and digital literacy and an understanding of the world around us by integrating mathematics with other areas.
Spain	The curriculum emphasizes the development of social skills to meet the sustainable development goals outlined in the European agenda 2020. The non-governmental organization, *Ayuda en Acción* (2020), prepared the document called *education strategy for global citizenship 2030*, where the objectives are directly linked to the SDGs. It incorporates strategies to ensure that all students gain the necessary digital skills for employment and active participation in society, as well as strong social skills. The curriculum provides detailed descriptions for acquiring these skills. Teachers should utilize technologies, promote group work, and adopt collaborative learning projects to facilitate the acquisition of social and emotional skills. The curriculum plan includes a module aimed at building a personal itinerary for employability, lasting 50 hours, focusing on the development of interpersonal skills that contribute to the insertion of students into the job market.

whereas Spain focuses on employability preparation and alignment with the SDGs. Both curricula aim to shape active, entrepreneurial citizens equipped with 21st century skills. The Spanish curriculum, written in a more contemporary and European language, emphasizes developing interpersonal skills and lifelong learning, facilitating the transition into the labor market.

FINAL CONSIDERATIONS

The curriculum takes very different formats in the two countries studied in all categories of analysis. In Portugal, mathematics is an autonomous subject with a practical, applied approach that is integrated with other areas of knowledge. In contrast, in Spain, mathematics is incorporated into applied sciences and addressed within a broader context, aligned with other scientific disciplines. Whether the topics studied, with more advanced mathematics in Portugal, the structure and assessment are presented very differently. In the curricula analyzed, student performance assessment practices are centered on formative assessment and have a prominent place in the curriculum of both countries. However, its expression is qualitative in Spain and quantitative in Portugal.

The action and purpose of the curriculum are similar in both countries. Technicians are trained according to the national qualifications catalog, and students receive a double certification: academic (12th grade of secondary level of education in Portugal and a 10th grade of secondary level of education in Spain, in both countries the end of compulsory education) and professional.

Perhaps the most similar is the focus on developing students' skills and the link to the *students' profile by the end of compulsory schooling*, a document in both countries. However, even in this sense, the Spanish curriculum adopts a more current language, aiming to develop interdisciplinary skills and, in the case of mathematics, STEM skills in students.

Due to the abundance of training areas available in both countries, it was not possible to ascertain the existence of a link between the contents of the mathematics curriculum and the students' professional profiles. Instead, there is a common interest in equipping students with tools that enable them to learn throughout their lives and acquire the necessary skills to become active and informed citizens. Nevertheless, we do not observe a significant emphasis on mathematical training that serves the professional aspects of the courses within the Spanish curriculum, apart from some indications that such a relationship should be established. In contrast, this intent is clearly evident in the Portuguese curriculum, which permits schools and teachers to select the modular plan for the mathematics discipline, comprising 50% of the course load, according to the interests of the class group and the geographical location of the school.

Both in the area of knowledge-based skills and the area of skills based on self-concept, both curricula analyzed present important convergences like the diversification of mathematics themes in vocational courses and some methodological suggestions that support teachers' work. Both curricula also intend to contribute to developing students' personal and interpersonal schools, according to the *students' profile by the end of compulsory schooling,* a document that exists in Portugal and in Spain. Due to what is described above, which allow us to infer a set of recommendations for the vocational course program:

(1) diversification of the mathematical themes worked on, covering analysis, algebra, geometry, probabilities, and statistics,

(2) definition in the curriculum of guidelines for the teaching and learning of mathematics, transversal to all themes,

(3) contribution to the development of transversal skills that allow lifelong learning, and

(4) appreciation of the importance of teachers appropriating the curriculum guidelines, which will be facilitated with the creation of resources and access to training.

It is important to highlight that both curricula aim to develop active and entrepreneurial citizens equipped with 21st century skills, focusing on developing social, civic, and interpersonal competencies. In this regard, it is suggested that, in Portugal, there could be a greater emphasis on the development of interpersonal skills, as well as a more explicit alignment with the SDGs. Similarly, in Spain, it would be beneficial to integrate Education for Citizenship more explicitly across all areas of the curriculum, thereby reinforcing civic education and students' social commitment.

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