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Mediating role of metacognitive awareness between attitude and mathematics reasoning in pre-service teachers

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ABSTRACT

This study aimed to determine the relationship between attitude, metacognitive awareness, and mathematics reasoning, as well as investigate the role of metacognitive awareness as a mediator. This study examined data from 378 pre-service teachers in Malaysia. The data were gathered by administering questionnaires and a mathematics reasoning assessment. The data were examined using the structural equation modelling technique. The results demonstrated substantial correlation through all variables, and path analysis revealed that metacognitive awareness mediates the relationship between attitude and mathematics reasoning. The results indicate that attitude has an essential impact on influencing the reasoning capabilities of pre-service teachers in mathematics by fostering the growth of metacognitive awareness.

Keywords: attitude, mathematics reasoning, metacognitive awareness, pre-service teachers

INTRODUCTION

Mathematics metacognitive awareness is a crucial component in the education of pre-service teachers, particularly in enhancing their teaching efficacy and improving student outcomes. Metacognitive awareness refers to the knowledge and regulation of one's cognitive processes, which are essential for effective problem-solving and learning in mathematics. Research indicates a significant relationship between metacognitive awareness and pre-service teachers' beliefs about mathematics, which in turn influences their teaching practices and their students' mathematical success (Hassan & Rahman, 2017).

In addition, the development of metacognitive awareness is linked to higher-order thinking skills, which are essential for mathematical reasoning and problem-solving. Mathematical reasoning equips pre-service teachers with the ability to analyze and justify mathematical arguments, which is crucial for effective teaching (Mata-Pereira & da Ponte, 2017). Mata-Pereira and da Ponte (2017) also highlight that teacher actions significantly influence students' reasoning processes, emphasizing the need for pre-service teachers to engage in practices that promote generalization and justification in mathematics (Manmai et al., 2021). Previous studies suggested that fostering reasoning habits such as problem analysis and strategy implementation is vital for developing students' mathematical understanding. Therefore, pre-service teachers must cultivate reasoning skills to model and encourage similar skills in their future students (Manmai et al., 2021).

Lastly, the importance of fostering a positive attitude towards mathematical reasoning is underscored by the impact of teacher attitudes on student learning outcomes. Zsoldos-Marchis (2015) emphasizes that teachers' attitudes towards mathematics significantly influence their students' attitudes and performance. When pre-service teachers exhibit enthusiasm and a cheerful disposition towards mathematical reasoning, they are more likely to inspire similar attitudes in their students, creating a supportive and encouraging classroom environment.

Research Problem

One prominent area for improvement is the greater understanding of metacognitive strategies and pre-service teachers' application in teaching practices. Ekici et al. (2019) emphasize that while metacognition is crucial for successful learning, many pre-service teachers exhibit low metacognitive awareness levels. This lack of awareness can hinder their ability to reflect on their thinking processes and implement effective metacognitive strategies in their classrooms. Furthermore, Bulut (2018) stated that teachers often possess inadequate metacognitive skills, leading to ineffective teaching practices and a failure to foster similar skills in their students.

Another area for improvement is the development of mathematical reasoning skills among pre-service teachers. Morali and Filiz (2023) found that many pre-service mathematics teachers must improve their mathematical noticing skills, which are essential for recognizing and interpreting mathematical concepts and student reasoning. This lack of noticing skills can lead to difficulties in understanding students' thought processes and effectively addressing misconceptions. Similarly, Murtafiah et al. (2018) emphasize that pre-service teachers often need help to explain mathematical concepts clearly, particularly when addressing students' misconceptions. This inability to articulate reasoning can further complicate the learning process for students.

Lastly, the critical issue is the prevalence of traditional beliefs about mathematics among pre-service teachers. Segarra and Julia (2022) found that many pre-service teachers enter their training programs with conventional views of mathematics, which often emphasize rote memorization and procedural knowledge over conceptual understanding. This reliance on traditional beliefs can limit their ability to engage students in meaningful mathematical reasoning and problem-solving. Similarly, Sopekan and Awofala (2019) highlights that pre-service teachers often hold mixed beliefs about mathematics, which can lead to consistency in their teaching practices and hinder their ability to foster a positive learning environment.

Research Objectives

This study aimed to determine the relationship between attitude, metacognitive awareness, and mathematics reasoning, and investigate the role of metacognitive awareness as a mediator.

Research Questions

Research questions based on this study focus:

1. What is the relationship between pre-service teachers' attitudes and mathematical reasoning?
2. Does metacognitive awareness influence the relationship between attitude and mathematical reasoning among pre-service teachers?
3. How does metacognitive awareness mediate the relationship between pre-service teachers' attitudes and their mathematical reasoning abilities?

Research Hypotheses

Below are the hypotheses of this research:

Hypothesis 1: A significant relationship between pre-service teachers' attitudes and mathematical reasoning.

Hypothesis 2: A significant relationship between pre-service teachers' attitudes, metacognitive awareness, and mathematical reasoning.

Hypothesis 3: Metacognitive awareness mediates the relationship between pre-service teachers' attitudes and their mathematical reasoning abilities.

Attitude toward Mathematics Reasoning

Numerous factors impact undergraduates' views toward mathematics as they advance. According to Mazana et al. (2018), undergraduates initially tend to view mathematics positively, but this positivity might decrease as they move on to higher educational levels. According to Camacho (2022), undergraduates with a positive attitude toward mathematics tend to perform better in their mathematics courses. Moreover, the specific sorts of mathematical reasoning necessary for university mathematics assessments have the potential to influence and develop undergraduates' comprehensive mathematical reasoning skills (Bergqvist, 2007).

Undergraduates' attitudes toward mathematics encompass psychological aspects that substantially impact their academic achievement. Research has indicated that undergraduates' perceptions and beliefs about mathematics significantly influence academic success, with positive attitudes potentially resulting in enhanced mathematics performance (Alibraheim, 2021). The affective domain in mathematics education, containing attitudes, emotions, and beliefs, substantially influences undergraduates' experiences and achievements in mathematics (Kashefi et al., 2017) Beliefs and cognitive processes significantly shape attitudes toward mathematics, illustrating the intricate relationship between emotional and cognitive factors in undergraduates' perspectives (León-Mantero et al., 2020).

The cognitive aspects influencing undergraduates' beliefs and thoughts about mathematics must be considered to fully understand their attitudes about the subject in the cognitive domain (Roick & Ringeisen, 2017). Studies reveal that beliefs and cognitive processes are among the cognitive elements that impact attitudes toward mathematics (Mata-Pereira & da Ponte, 2017). Zeleke and Semela (2017) highlight the significance of cognitive dimensions in comprehending undergraduates' attitudes toward particular disciplines, and they have developed an attitude measure to evaluate attitudes toward mathematics.

The behavioral aspect of undergraduates' attitudes toward mathematics refers to the observable activities and responses associated with their involvement in the subject. Studies suggest that undergraduates' attitudes toward mathematics are impacted by their behaviors, including their degree of engagement in mathematics related activities and problem-solving strategies (Albay, 2019; Kashefi et al., 2017). Similarly, the influence of negative experiences in mathematics instruction on pre-service teachers' mathematics anxiety highlights the observable expressions of their attitudes toward mathematics (Zulkipli et al., 2020). Behavioral indications reflect their attitude toward mathematics, such as undergraduates' willingness to seek assistance and participate in cooperative learning (Salam et al., 2020).

Metacognitive Awareness and Attitude

Metacognitive awareness and attitude towards mathematics are essential in determining undergraduates' academic achievement. Ajisuksmo and Saputri (2017) established a significant relationship between individuals' attitudes toward mathematics, their metacognitive awareness, and their achievements in mathematics. According to Izadi et al. (2018), undergraduates with a good attitude toward mathematics and a strong understanding of their reasoning processes to achieve higher scores in mathematics related assignments. In addition, studies have demonstrated that employing metacognition-based tactics can improve undergraduates' metacognitive awareness and mathematical abilities (Salam et al., 2020). Metacognitive awareness is crucial for achieving academic success, nurturing positive character qualities, and encouraging the development of learner personality traits (Suryadi & Santoso, 2017).

Metacognitive awareness, particularly declarative knowledge, is crucial for undergraduates' mathematical success (Roick & Ringeisen, 2018). Research has shown that metacognitive knowledge, which includes declarative, procedural, and conditional knowledge, is essential for effectively controlling cognitive activities, especially in mathematical problem-solving contexts (Radmehr & Drake, 2020).

Declarative knowledge enables undergraduates to clearly understand mathematical concepts, rules, and strategies, empowering them to make informed decisions about their problem-solving processes (Radmehr & Drake, 2020). Additionally, metacognitive awareness, including declarative knowledge, has been associated with improved academic motivation and achievement among undergraduates (Abdelrahman, 2020).

Procedural knowledge, a part of metacognitive awareness, refers to effectively carrying out processes (Mitsea & Drigas, 2019). According to Wonu and Paul-Worika (2019), empirical evidence suggests that enhanced metacognitive knowledge, encompassing procedural knowledge, can be achieved by implementing suitable techniques.

It is essential to realize that conditional knowledge refers to the ability to understand when and why to apply particular strategies or approaches in problem-solving, which is relevant to explaining mathematics metacognitive awareness among undergraduates (Wonu & Paul-Worika, 2019). Research has indicated that metacognitive strategy education enhances metacognitive awareness, especially about conditional knowledge (Robillos & Bustos, 2022). Undergraduates with higher conditional knowledge can use appropriate problem-solving techniques more effectively (Wonu & Paul-Worika, 2019).

Planning is a crucial element of metacognitive awareness in mathematics for undergraduates. It involves the ability to devise strategies and arrange one's approach to problem-solving (Adinda et al., 2021). An essential aspect of mathematics is the implementation of effective planning, which involves establishing clear objectives, delineating the necessary actions to attain them, and proactively considering potential obstacles that may arise during the problem-solving process. Teachers can enhance undergraduates' metacognitive planning skills by implementing interventions emphasizing the significance of creating goals, analyzing tasks, and selecting strategies (Salam et al., 2020). By implementing reflective planning strategies, instructors can support undergraduates in developing a more deliberate and strategic approach to solving mathematics problems.

Studies have shown the significance of metacognitive monitoring in enhancing undergraduates' problem-solving abilities and mathematical performance (Lingel et al., 2019). By consistently monitoring their cognitive processes and techniques, undergraduates can identify flaws, misunderstandings, or inefficiencies in their problem-solving techniques, resulting in improved results in mathematics assignments. Additionally, metacognitive monitoring facilitates the relationship between problem-solving abilities, metacognitive awareness, and academic achievement (Hassan & Rahman, 2017) Undergraduates with extensive monitoring skills have a greater capacity to control their cognitive processes, leading to improved problem-solving abilities and increased mathematics achievement levels.

Studies have demonstrated that metacognitive evaluation significantly enhances undergraduates' problem-solving abilities and academic performance in mathematics (Abdelrahman, 2020). By engaging in the critical analysis process of their problem-solving strategy, undergraduates' can identify specific areas that require improvement, correct faults, and enhance their approaches to attain superior results in solving mathematical problems. Also, evaluating metacognition is vital in fostering undergraduates' ability to regulate their learning and knowledge of their thinking processes. Undergraduates can better comprehend their cognitive aptitudes and limitations by deliberate introspection and evaluating their problem-solving methods, improving their metacognitive abilities and overall academic achievement (Abdelrahman, 2020).

Mathematics Reasoning and Metacognitive Awareness

For academic success, undergraduates must possess excellent mathematical reasoning skills and metacognitive awareness. Researchers investigated the correlation between undergraduates' metacognitive awareness and mathematics reasoning (Adinda et al., 2021; Tak et al., 2022). They found undergraduates can distinguish their metacognitive awareness when engaging in mathematical reasoning problems (Morsanyi et al., 2018). Beyond that, there is a direct relationship between metacognitive awareness and mathematics thinking in undergraduates. Meanwhile, a correlation has been seen between metacognitive awareness and self-efficacy in mathematical reasoning, indicating an association between improved mathematical reasoning abilities and higher metacognitive awareness (Robillos & Bustos, 2022).

The literature review identified several factors contributing to weaknesses in mathematical reasoning among university undergraduates (Agustyaningrum et al., 2019; Jensen et al., 2017). Studies have shown that undergraduates frequently face challenges in mathematical reasoning because they need help employing appropriate problem-solving frameworks, recalling prior information, and establishing links between the physical and abstract aspects of mathematical thinking (Woolley et al., 2018). The disparity between secondary school and university mathematics has been recognized as a notable concern since undergraduates must be more adequately equipped with fundamental mathematical abilities and subject-specific skills (Harding et al., 2011). The deficiency in mathematical reasoning skills has been emphasized, as evidenced by pupils' low scores on mathematical reasoning assessments (Singh et al., 2020).

For undergraduates, mathematical reasoning is essential to developing critical thinking skills and improving their problem-solving ability, especially in solving complicated problems and making well-informed judgments. Critical thinking requires properly analyzing, evaluating, and understanding information. Critical thinking abilities are essential in mathematics education for learners to approach mathematical problems with logic, reasoning, and creativity (Tak et al., 2023). In addition, Herizal et al. (2022) mention a strong link between critical thinking and mathematics education, implying that every aspect of mathematics instruction helps undergraduates develop critical thinking abilities. The finding highlights how critical thinking exercises should be incorporated into mathematics instruction to assist undergraduates in developing their analytical and evaluative skills.

Undergraduates' ability to reason mathematically is greatly influenced by geometry, which calls for combining cognitive skills and problem-solving techniques. Undergraduates must use deductive reasoning, logical reasoning, and spatial visualization to analyze shapes, angles, and spatial relationships in geometry tasks (Chan et al., 2021). Masfingatin et al. (2020) highlight the value of creative reasoning abilities in solving geometric difficulties, stressing the necessity of creative thinking and a clear understanding of geometric ideas.

Among undergraduates, mathematics comprehension is based on their ability to use mathematical reasoning when dealing with fractions. Undergraduates require critical thinking and reasoning to understand and utilize fraction-magnitude knowledge for numerical understanding in general. Furthermore, Braithwaite et al. (2019) highlight the difficulties many undergraduates' face when learning fraction arithmetic, which can obstruct their progress in mathematics. Previous studies highlight the significance of developing mathematical reasoning abilities, especially in fractions, to navigate complex mathematical ideas effectively (Braithwaite et al., 2019) .

The need to improve pupils' mathematical reasoning skills to support the shift to algebra is highlighted by research conducted by Singh et al. (2020). To help undergraduates understand algebraic ideas and reason effectively, they must acquire solid basic skills. The ability to evaluate, decipher, and work with algebraic expressions and equations is known as algebraic reasoning, and it is necessary for working through mathematical problems. In addition, Basir et al. (2022) illustrate how undergraduates transition from arithmetic to algebra when they work on numerical tasks and generalize by discussing algebraic thinking inside Marzano's taxonomy cognitive system. This change emphasizes how important undergraduates are to express their justifications and calculations in symbolic language, demonstrating a deep comprehension of algebraic ideas (Basir et al., 2022).

METHODOLOGY

This study comprised 378 pre-service teachers from a public institution situated in one of Malaysia states. The participants were chosen through a multistage stratified random sampling method. Once they graduate from their training institute, the selected participant will be served in elementary and secondary school. This research selected first year students, which is advantageous due to their recent experiences in the secondary education system and provides fresh memories and insights into their educational transitions.

The methodological paradigm of research using quantitative methods, particularly survey methods and structural equation modeling (SEM), is grounded in the philosophy of positivism (Awang, 2018). This paradigm emphasizes the collection and analysis of numerical data to understand relationships between attitude, metacognitive awareness and mathematics reasoning.

As defined by Chua(2020), quantitative research is characterized by its reliance on structured instruments for data collection, such as surveys or questionnaires administered to a sample population. This method allows researchers to gather data that can be statistically analyzed to conclude the broader population. Chua (2020) highlights the effectiveness of survey methods in capturing the characteristics of a large population, ensuring accurate results that facilitate informed decision-making. In this research, surveys are designed to collect structured responses, often using Likert scales or multiple-choice questions, which lend themselves to quantitative analysis.

This research utilized a survey method to assess the usability of attitude, metacognitive awareness, and mathematics reasoning assessment, adapting existing instruments to meet the study's specific objectives. The survey included a five-point Likert scale, which enabled participants to express their level of agreement with various statements, thus providing quantifiable data for analysis.

Research Instrument

The adaptation of instruments is crucial in ensuring that the survey accurately reflects the constructs studied. These studies demonstrated the effectiveness of using adapted instruments to measure pre-service teachers attitudes and metacognitive awareness towards mathematics reasoning. By modifying existing scales, researchers can tailor the questions to better fit their participants' cultural and educational context, thereby improving the relevance and applicability of the data collected. The selection of measurement instruments in research is a critical process that requires careful consideration of various factors, including psychometric properties, relevance to the target population, and the study specific objectives (Chua, 2020). The participants cannot justify the selection of these instruments because instruments in these studies over others with similar characteristics, it is essential to evaluate the validity, reliability, and responsiveness, as highlighted in the literature.

This research also has got six expert validations in ensuring the instruments meet the necessary standards. The validation involves evaluating whether the instrument covers the relevant content area comprehensively, and engaging experts in the field to review the instrument can enhance content validity. Besides that, this research utilized confirmatory factor analysis (CFA) to validate a competence model for educational researchers, demonstrating the application of construct validity in research. The construct validity assesses whether the instrument truly measures the theoretical construct it claims to measure. Besides that, the validation can be evaluated through factor analysis, which examines the relationships between items and the underlying factors they represent (Chua, 2020).

Attitude toward mathematics

A survey of 17 items was adapted to collect data on pre-service teachers' attitudes towards mathematics. Seventy entries were derived from Fennema and Sherman (1976) and Tapia and Marsh (2004). The items were associated with affective, behavioral, and cognitive learning dimensions. The results of factor analyses were utilized to evaluate the validity and reliability of this instrument. The factor loading was retrieved by exploratory factor analysis, aligning with the components of attitude. Also, CFA showed that the average variance extracted (AVE) was 0.612, and construct reliability (CR) obtained ranged from 0.713 to 0.861, respectively. Therefore, the AVE for each construct was above acceptable level of 0.50, and CR was above the acceptable level of 0.70, according to suggestion by Awang (2018).

Metacognitive awareness

The researchers adapted the metacognitive awareness inventory, created by Rahman et al. (2014) and Schraw and Dennison (1994), to collect data on score metacognitive awareness from undergraduates. The items were classified based on the six subcomponents of metacognitive awareness proposed by Rahman et al. (2014) and Schraw and Dennison (1994), which include declarative knowledge, procedural knowledge, conditional knowledge, planning, monitoring, and assessment. The previous study by Rahman et al. (2014) reported the validity, reliability, and applicability of applying this inventory in the Malaysian education setting. Meanwhile, CFA showed that the AVE obtained was 0.550, and CR ranged from 0.751 to 0.812, respectively. Consequently, the AVE for each construct was above acceptable level of 0.50, and CR was above acceptable level of 0.70, according to suggestion by Awang (2018).

Mathematical reasoning

Pre-service teachers' mathematics reasoning assessment scores were used to collect data on their mathematics reasoning abilities. The reasoning examinations consist of critical thinking, algebraic reasoning, geometry, and fractions. The scores were derived from the rubric, validated, and standardized by experts to

Table 1. Correlation among variables

Variables	β	p
Direct model		
Attitude → Mathematics reasoning	0.83	0.001
Mediating model		
Attitude → Mathematics reasoning	0.58	0.001
Attitude → Metacognitive awareness	0.65	0.001
Metacognitive awareness → Mathematics reasoning	0.38	0.001

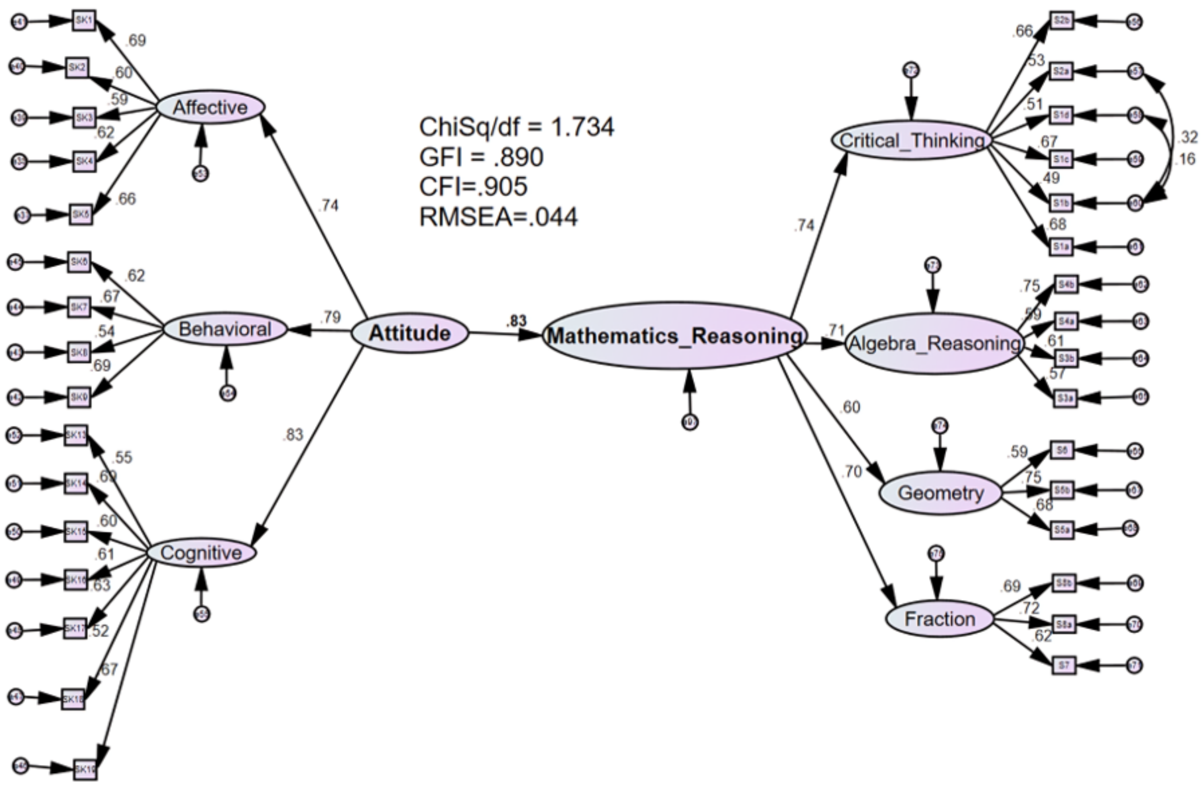

Figure 1. The direct effect of attitude toward mathematics reasoning (the authors' own work)

ensure consistency. The critical thinking questions were adapted from Morris (2010); algebraic reasoning questions were adapted from Jeffrey and William (2018) and Larkin and Jorgensen (2015); geometry questions were adapted from Aydin and Halat (2009); and fraction questions were adapted from Ensley and Crawley (2006). Further, CFA showed that the AVE obtained was 0.561, and CR ranged from 0.790 to 0.807, respectively. The AVE for each construct was above acceptable level of 0.50,and CR was above acceptable level of 0.70 according to suggestion by Awang (2018).

FINDINGS AND DISCUSSION

The analysis revealed a significant direct relationship between attitude and mathematics reasoning, with a path coefficient of 0.83 at a significance level of $p < 0.001$ (**Table 1**). This finding indicates that attitude has a positive and statistically significant direct impact on the reasoning abilities of pre-service mathematics teachers. The calculated coefficient of determination (R^2) was 0.58, indicating that attitude accounted for 58% of the variability in mathematics reasoning. **Figure 1** displays the findings of the exploratory analysis.

The analysis of the mediation model revealed that attitude continued to have a positive and significant direct impact on mathematics reasoning even after metacognitive awareness was used as a mediator. The magnitude of this effect fell to 0.58. The decrease value identified in this study supports the idea that metacognitive awareness mediates the relationship between attitude and pre-service mathematics teacher's reasoning.

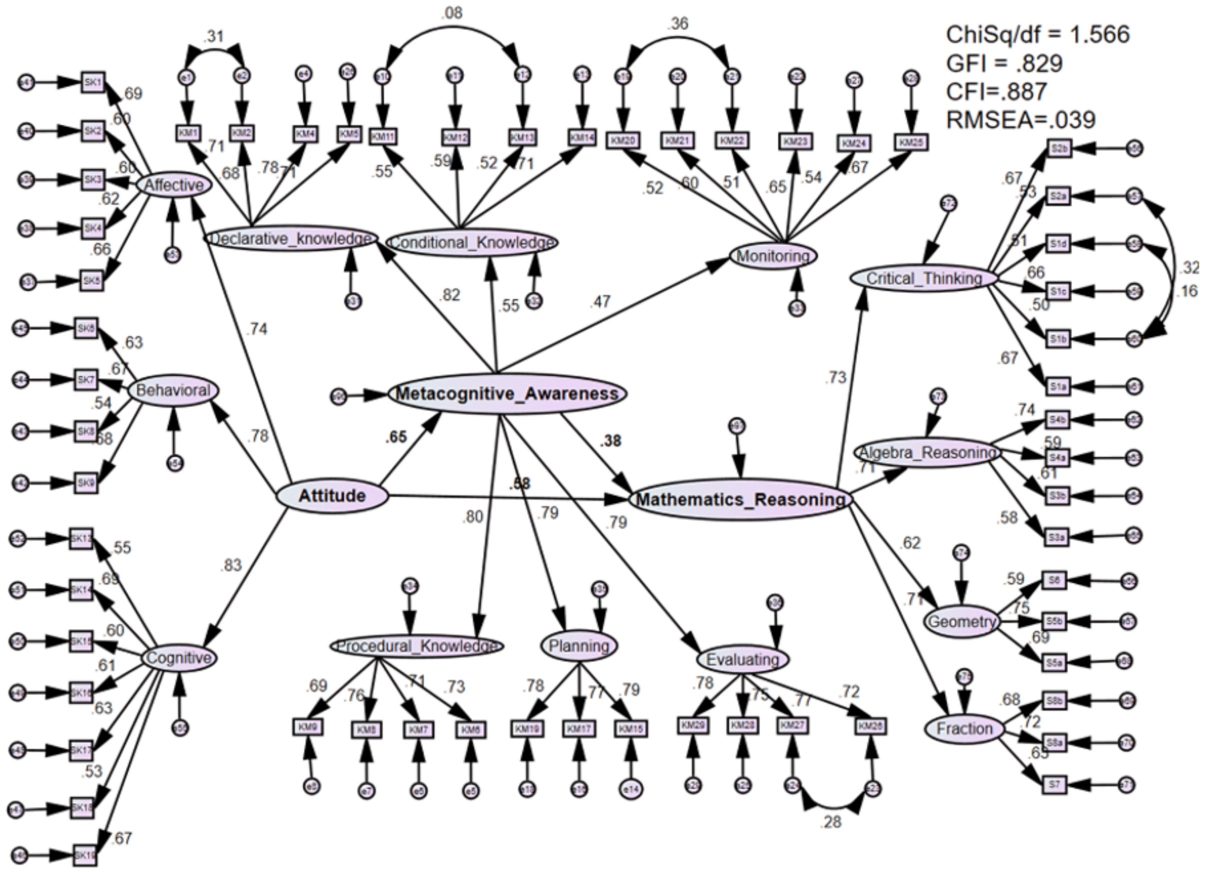

Figure 2. Analysis of the mediation model (the authors' own work)

From **Figure 1**, the path coefficient was examined to determine significance of the relationships between attitude and mathematics reasoning. The result indicated that attitude had a significant positive effect on mathematics reasoning. ($\beta = 0.83$, $p < 0.01$), supporting hypothesis 1. The path coefficient for attitude dimensions (affective = 0.74; behavioral = 0.79; cognitive = 0.83). The factor loading for constructs of attitude ranged from 0.49 to 0.69. Meanwhile, constructs for mathematics reasoning ranged from 0.51 to 0.75. The path coefficient in the model indicated that higher attitude optimized mathematics reasoning.

The SEM analysis in **Figure 2** indicated a good fit for the proposed model, with the following fit indices degree of freedom Chi-square (df/χ2) = 1.566; $p < 0.01$, goodness fit index = 0.829, comparative fit index = 0.887, and root mean square error of approximation = 0.039. The results indicated that the model in these findings adequately represent the data and confirming the hypothesized relations among the constructs and validating hypothesis 2.

Additionally, the mediation effect of metacognitive awareness on the relationship between attitude and mathematics reasoning was tested. The analysis revealed a significant indirect effect ($\beta = 0.58$, $p < 0.01$), indicating that metacognitive awareness mediates the relationship between attitude and mathematics reasoning, supporting hypothesis 3. This finding highlights the importance of metacognitive awareness as a pathway through which high attitude impacts mathematics reasoning abilities among pre-service teachers.

The findings show a positive correlation between attitude and mathematics reasoning, which aligns with previous research. Mazana et al. (2018) found that undergraduates initially tend to view mathematics positively, but this positivity might decrease as they move on to higher educational levels. Meanwhile, Camacho (2022) found that undergraduates with a positive attitude toward mathematics tend to perform better in their mathematics courses.

The study indicated a positive and statistically significant correlation between metacognitive awareness and mathematics reasoning. These results align with the findings of earlier research. Adinda et al. (2021) found a correlation between undergraduates' metacognitive awareness and mathematics reasoning. Morsanyi et al. (2019) found that undergraduates can distinguish between their metacognitive awareness when engaging in mathematical reasoning problems. In addition, Kartika (2019) found a direct relationship between metacognitive awareness and mathematics thinking among undergraduates. Besides that, Robillos and Bustos (2022) found a correlation between metacognitive awareness and self-efficacy in mathematical reasoning, indicating an association between improved mathematical reasoning abilities and higher metacognitive awareness.

In findings of this studies further supported a positive correlation between attitude and metacognitive awareness. The results of earlier investigations are parallel to those of the present investigation. Ajisuksmo and Saputri (2017) found a significant relationship between individual attitudes toward mathematics, metacognitive awareness, and achievements in mathematics. Meanwhile, Izadi et al. (2018) found that undergraduates with a good attitude toward mathematics and a strong understanding of their reasoning processes will likely achieve higher scores in math-related assignments. In addition, Salam et al. (2020) found that employing metacognition-based strategies can improve undergraduates metacognitive awareness and mathematical ability. Likewise, Suryadi and Santoso (2017) found that metacognitive awareness is crucial for achieving academic success, nurturing positive character qualities, and encouraging development of personality traits in learners.

Improving metacognitive awareness among pre-service teachers can be achieved through targeted instructional practices, integration of problem-posing activities, and the promotion of collaborative learning environments. These strategies enhance metacognitive skills and contribute to the developing a positive attitude toward mathematics and improved reasoning abilities. By focusing on these areas, educators can better prepare future teachers to foster similar skills in their own students, ultimately leading to a more effective mathematics education.

Implication of Research

This research is important in educational research, where understanding trends and patterns can inform policy decisions. The implications of these studies findings for educational institutions in Malaysia suggest that this quantitative analysis provides a robust foundation for advocating changes in professional standards and support systems for mathematics educators.

For instance, the findings of this study point out that understanding the challenges faced by educated teachers can inform teacher education programs, highlighting the need for further studies that explore the experiences of diverse teacher populations. The findings of these studies can lead to more inclusive and supportive educational environments. Therefore, this research provides valuable insights that can inform educational practices, influence policy decisions, and guide future research endeavors. By leveraging the strengths of this methodological paradigm, researchers can contribute to the advancing knowledge in education and improving of teaching and learning outcomes.

CONCLUSIONS

The results of this study have significant educational consequences, particularly in the higher education setting. The findings indicate that attitude and metacognitive awareness significantly enhance pre-service teachers' mathematical reasoning. Hence, it is crucial to focus on cultivating attitude and metacognitive awareness to enhance pre-service teachers mathematical reasoning abilities. This study also presented evidence of metacognitive awareness as a mediator in the correlation between attitude and mathematics reasoning. This discovery suggests that cultivating attitude can be improved pre-service teachers' metacognitive awareness, resulting in improved academic success. The limitation of this study utilized random sampling methods that may not adequately represent the broader population of pre-service teachers from private institute in Malaysia. Therefore, future studies should aim for more diverse samples that include various gender, ethnic, and educational backgrounds to enhance the applicability of the results.

In conclusion, SEM in these studies addresses critical research gaps by enabling the exploration of complex relationships among latent and observed variables across various domains. The model of these studies' ability to mediate, interact with effects, and multiple constructs simultaneously allows researchers to derive richer insights and contribute to theoretical advancements in Malaysian education.

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Ethics declaration: This study was approved by the Research Ethics Committee at the Universiti Malaya on 3 June 2022 with approval number UM.TNC2/UMREC_1907. Informed consent was obtained from participants. Comprehensive information about the study was given to the participants ensuring that their participation was voluntary. Sensitive personal data was handled with the utmost care and confidentiality. All identifying information was removed from data so that data could not be linked back to an individual. Data was stored in encrypted formats on secure servers with restricted access and only the author was authorized to access. No identifiable information was included in the article.

Declaration of interest: The authors declared no competing interest.

Data availability: Data generated or analyzed during this study are available from the authors on request.

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