European Journal of Science and Mathematics Education, 2026, **14**(3), 427-448
ISSN: 2301-251X (Online)

OPEN ACCESS

Research Article

Pedagogical intervention with a mathematically gifted student–Expanding problem-solving strategies and developing metacognition: A case study

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Citation: Panáčová, J., & Veseláková, J. (2026). Pedagogical intervention with a mathematically gifted student–Expanding problem-solving strategies and developing metacognition: A case study. *European Journal of Science and Mathematics Education, 14*(3), 427-448. https://doi.org/10.30935/scimath/18785

ARTICLE INFO

Received: 21 Oct 2025

Accepted: 17 May 2026

ABSTRACT

The topic of gifted students has been significantly reflected in a number of scientific publications in recent years, many of which point to the fact that the giftedness of these students is not always apparent in the school environment, and therefore not always identified (Siegle et al., 2025). This article presents a theory corresponding to this topic and related research that deals with a case study using qualitative research methods. The selected case study focuses on a student who, before entering primary education and during it, appeared to be mathematically gifted at the home environment, but his giftedness was not identified in the school environment, let alone further developed. The insufficient fulfilment of the student's development needs in mathematics within the school environment led to the search for individual support outside the school environment. Based on this request, the student was involved in a one-year pedagogical intervention in mathematics. The article presents typical manifestations of the student's mathematical giftedness and describes the process of this individual intervention with the student, which aimed to expand his strategies for solving mathematical problems and develop his metacognition. Metacognition is crucial not only for academic performance but also for the autonomy, self-confidence and socio-emotional well-being of gifted students. The study therefore highlights the importance of supporting metacognitive thinking as a key tool that will enable gifted learners to face challenges more effectively, adapt to new situations and fully realize their potentials.

Keywords: mathematically gifted student, pedagogical intervention, development of giftedness, metacognition, case study, problem-solving strategies

INTRODUCTION

Gifted children represent a group of learners with exceptional potential which, however, does not always develop automatically (Renzulli, 1978; Subotnik et al., 2011). Their abilities often exceed the standard expectations of school, which can lead either to the flourishing of their potential or, conversely, to frustration and a lack of motivation in learning. Research has shown that gifted students have specific educational needs–they require an environment that is intellectually stimulating, provides space for independent discovery, and supports the development of cognition (Yanık & Afat, 2022).

One of the key elements that can help them to realize their full potential is metacognition. Studies have found that gifted students often possess stronger declarative metacognitive knowledge and a greater ability to transfer strategies to new situations, yet they do not always spontaneously demonstrate better use of strategies or systematic monitoring of their own learning (Carr et al., 1996; Tibken et al., 2022). A number of studies (e.g., Knox, 2017; Portešová & Veenman, 2021) confirm that metacognitive development can be supported through pedagogical intervention and through guided discussion with the student about different ways of solving mathematical problems.

The development of metacognitive strategies is particularly important because even gifted students may encounter limits to their intuitive approaches to learning. When they learn to reflect on how they learn, they are better equipped to deal with difficult tasks, adapt to new situations, and overcome obstacles (Oppong et al., 2019; Schraw & Graham, 1997). Metacognitive skills contribute not only to academic achievement, but also to autonomy, self-confidence, and socio-emotional competence (Sastre-Riba, 2011; Yanık & Afat, 2022). Moreover, longitudinal studies show that procedural metacognition–the ability to actively apply strategies–has a significant impact on the academic success of gifted students, even beyond the role of intelligence itself (Tibken et al., 2022). These findings underline the importance of systematically developing metacognitive awareness among gifted learners, opening the way to their long-term academic and personal growth.

Although gifted students often demonstrate superior declarative knowledge of learning strategies and the ability to transfer them, their spontaneous use and systematic monitoring of learning is not consistently higher than that of the general population (Carr et al., 1996; Tibken et al., 2022). This discrepancy represents a significant gap in both research and practice–while the intellectual abilities of gifted students are well documented, their metacognitive development and its systematic support remain under-explored. In response to this under-researched area, a study was conducted to examine to what extent and in which areas systematic support implemented through pedagogical intervention with a mathematically gifted student could influence his choice of problem-solving strategies and enhance his metacognitive development. The results of this research are presented in the following case study.

The aim of this study was to capture key phenomena related to the development of metacognition in a mathematically gifted student during a one-year pedagogical intervention, and to examine how the intervention influenced his use of problem-solving strategies and his metacognitive growth. The research was guided by two central questions:

1. What characteristics define a mathematically gifted student's approach to solving mathematical problems?
2. What forms of metacognitive development can be identified after a year-long intervention?

THEORETICAL BACKGROUND

Manifestations of a Mathematically Gifted Student

Given that this case study focuses on a student diagnosed with mathematical giftedness, the following section takes a more detailed look at the typical manifestations of this giftedness. Mathematically gifted students are those who are able to formulate problems spontaneously and who display strong abilities in generalization and abstraction. They possess exceptional memory, the ability to solve problems in unexpected ways, and they readily identify relationships and patterns. Such students often experience joy when presented with original problems and challenges in mathematics. They tend to learn quickly and can concentrate on a task for extended periods of time (Budínová, 2018; Shin & Oh, 2024). Mathematically gifted students are described as learners with advanced metacognitive abilities, creative mathematical thinking, and an exceptional capacity for problem-solving (Leikin, 2020; Leikin et al., 2017).

According to Sriraman (2003), gifted students in mathematics usually think differently from their peers; their reasoning is more complex, and they tend to perceive problems within a wider context. For such students, solving a mathematical problem may seem trivial, yet they often struggle to explain their reasoning to others or to record their thought processes in a way that others can understand (Koshy et al., 2009).

Mathematically gifted learners are often able to connect their existing knowledge with new concepts and apply it across different fields. Their approach to work is characterized by high flexibility and the ability to adapt strategies based on the nature of the task (Leikin, 2009), as well as by perseverance and enthusiasm, finding the process of problem-solving more rewarding than exhausting (Renzulli, 1978). Other researchers (Singer et al., 2016; Sipahi & Bahar, 2024) have observed that these students typically work quickly, can identify relationships between mathematical topics, and generate ideas and concepts without the need for teacher intervention. Singer et al. (2016) emphasize that mathematical giftedness does not always correlate with top performance; conversely, excellent results do not necessarily indicate mathematical giftedness. The assumption that mathematically gifted students always perform flawless calculations–and that computational proficiency automatically signals mathematical giftedness–may, according to Eraky et al. (2022), lead to misconceptions about mathematically gifted students.

Identifying Mathematical Giftedness

Budínová and Panáčová (2022) note that in regular primary and lower secondary school mathematics lessons, identifying a gifted child can be challenging. Some gifted learners find it difficult to explain how they arrived at a solution, which may lead teachers to interpret their behavior as a lack of understanding rather than evidence of giftedness. Such students often organize their notes and information in idiosyncratic ways that may appear chaotic to teachers. Omerović et al. (2020) argue that a student should be nominated as mathematically gifted based on their logical reasoning, independence in solving problem situations, high levels of abstraction, ability to generalize quickly, originality in problem-solving, and capacity to pose new questions and demonstrate critical thinking.

Mathematical giftedness can therefore be inferred from long-term observation of a student's abilities, their conceptual understanding, and from assessing the way they learn new procedures. A mathematically gifted student can be described as one who consistently demonstrates features of logical-mathematical intelligence, which is closely linked to the ability to generalize and solve problems with ease and insight.

According to Gardner (1983), logical-mathematical intelligence is the capacity to recognize the structure of a problem and to identify how it can be solved. Gardner (1983) states that this type of intelligence includes:

(a) the deductive application of rules and principles,

(b) numerical operations and the recognition of abstract patterns, and

(c) counting, sorting, and identifying relationships between phenomena.

The abilities of mathematically gifted students correlate with a range of personality traits such as a strong interest in mathematics, precision, critical thinking, perseverance, and curiosity (Omerović et al., 2020). These students often have broad interests, commonly combining mathematics with the natural sciences–physics, geography, chemistry–and sometimes with the social sciences, where mathematical thinking can also be applied.

From the perspective of understanding giftedness, Renzulli's (1978) *three-ring model* provides a valuable framework that can also be applied to mathematical giftedness. The model conceptualizes giftedness as the intersection of three components:

(1) above-average ability (general or specific);

(2) creativity;

(3) task commitment.

In this study, we base our observations of the manifestations of mathematical giftedness in a mathematically gifted student on this *three-ring model*, modified for mathematical giftedness (Schindler & Rott, 2017).

A Modified Version of Renzulli's (1978) *Three-Ring Model* for Mathematical Giftedness

Figure 1 presents Renzulli's (1978) *three-ring model* as modified for mathematical giftedness (Schindler & Rott, 2017). The model identifies three overlapping domains that together characterize mathematical giftedness:

1. *Above-average mathematical abilities*–such as strong logical and analytical thinking, rapid assimilation of mathematical concepts, exceptional numerical memory, and the ability to solve complex tasks without prior instruction.
2. *Creativity and intuition in mathematical problem-solving*–reflected in original approaches, flexibility of thought (for example, identifying multiple solutions to the same problem), and the capacity to generalize findings and formulate new hypotheses or strategies.

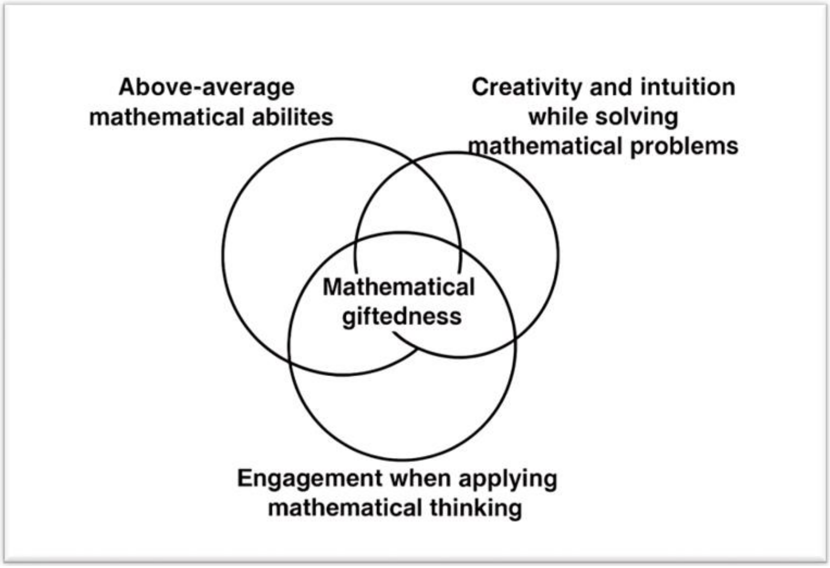

Figure 1. Renzulli's (1978) framework adapted for mathematical giftedness (Author's own work; inspired by Schindler & Rott (2017))

3. *Task engagement in applying mathematical thinking*–expressed through persistence when solving challenging problems, intrinsic motivation and enjoyment of mathematical work, and self-initiated mathematical activities such as solving problems beyond classroom requirements.

This modified *three-ring model* can assist teachers and psychologists in identifying mathematically gifted students not only based on performance, but also by considering their creative potential and motivation. It allows for a more nuanced understanding of mathematical giftedness and the design of differentiated approaches to teaching these learners.

Renzulli's (1978) framework provides a powerful lens for a holistic approach to mathematical giftedness. It helps educators recognize different types of potential in mathematically gifted students and develop these systematically in the classroom. With this model, teachers can act as facilitators of mathematical growth rather than mere transmitters of knowledge.

Metacognition in the Context of Mathematical Problem-Solving

Given that the presented research focused on the development of metacognition in mathematically gifted students through targeted intervention, the following section of the text is devoted to a detailed description of metacognition and its basic components. Flavell (1979) conceptualizes *metacognition* as an individual's capacity to plan, monitor and evaluate the methods they use when learning and acquiring knowledge. It can therefore be understood as an integral part of a learner's cognitive development. According to Veenman et al. (2006), metacognition involves knowing what one knows, evaluating the learning process, and managing it. Straka (2021) views metacognition as the ability to be aware of one's own cognitive processes–thinking, perception, memory, and so on–and to use this awareness when learning, problem-solving, or engaging in other cognitive activities. Scientific sources (Flavell, 1979; Lai et al., 2015; Straka, 2021; Veenman et al., 2006) present a three-component model of metacognition:

(1) *metacognitive knowledge*–this includes the knowledge a student has about cognitive abilities, primarily relating to their own mental processes; knowledge of general strategies that can be used to solve problems, and the conditions under which these strategies can be applied. This knowledge includes various aspects of learning, memory management, the effective storage of new information and its connection to existing knowledge, orientation in task-solving (which relates to the use of strategies), and the verbal description of the task solution after its completion;

(2) *metacognitive regulation*–the student actively manages own thinking, is able to judge the difficulty of a task; regulates strategies and modifies them if they are ineffective; verbally describes the process of

solving the task as it progresses; analyzes and corrects any mistakes; applies newly acquired knowledge; and

(3) *metacognitive monitoring*–the student continuously monitors own thinking; realizes whether he encountered this type of task before; recognizes the moment when "lost" in the task (*cognitive conflict*); identifies the mistakes and their causes.

Developing Metacognition in Gifted Students Through Intervention

Metacognition enables students to approach tasks more effectively. According to Budínová (2021), in mathematics it involves the ability to solve problems independently and to use strategies consciously. When students are familiar with a range of strategies, they can draw on this experience to make more informed and sophisticated choices. In mathematics, metacognition also manifests in the ability to verbalize and record each stage of the reasoning process rather than merely writing down the final result. For mathematically gifted students, this area can be particularly challenging, as they may find it difficult to articulate their thinking, either verbally or in writing. As mathematical tasks become more complex, a lack of metacognitive awareness can eventually slow the development of their potential.

Metacognition not only helps students solve problems correctly but also enables them to analyze their own thought processes and develop the ability to adjust their strategies (Moustakas & Gonida, 2023). Individuals with well-developed metacognitive skills demonstrate more flexible approaches, particularly when solving non-standard problem-solving tasks (Toikka et al., 2024; Xie et al., 2024).

Gifted students often reach correct solutions quickly and intuitively, but they may not share or explain their reasoning clearly to others. National Research Council [NRC] (2000) and Straka (2021) show that discussing different problem-solving strategies with students strengthens their metacognitive knowledge. Through such reflection, students gain new insights and can compare alternative approaches. In addition to obtaining the correct answer, the process of reasoning–verbal reflection and written representation–is of central importance. To support awareness of metacognitive processes and to encourage the use of a broader range of strategies, teachers can ask reflective questions such as: "How did you approach this problem?", "What other methods might you use to solve it?" If a student makes a mistake while solving a problem, the question "Where could you have gone wrong?" can help them realize the causes of their failure and, in the context of metacognition, understand their own thought process. In contrast, the question "How would you detect the mistake?" can subtly lead the student to understand the importance of checking for correctness as one of the stages of solving a task. During pedagogical intervention, it is appropriate to give gifted students more complex problems that do not have clear solutions, i.e., tasks with multiple solutions or tasks that have no solution. Students describe their procedures and discuss and evaluate the advantages and disadvantages of different strategies. Shared language and conversation about cognition and learning with the student helps them become aware of their own metacognitive knowledge as well as their own strategies for learning and thinking. According to Erdoğan (2025), systematic instruction and support for the development of metacognition can increase the effectiveness of problem-solving processes among mathematically gifted students. Schraw and Graham (1997) and Young and Worrell (2018) emphasized that planning, monitoring, and evaluation skills can be developed through instructional interventions, which presents a crucial opportunity to fully realize the potential of gifted individuals. These findings are consistent with the conclusions of Pintrich (2002), who states that pedagogical intervention can help students become more aware of their own thinking and develop their understanding of cognitive processes, which is related to the development of individual metacognitive components.

Pedagogical Intervention in Mathematics

Mathematical intervention focuses on developing students' problem-solving strategies and mathematical thinking through work on specific mathematical problems, drawing on classical problem-solving frameworks that emphasize strategic planning, execution, and reflection (e.g., Pólya, 1945). Pedagogical intervention, on the other hand, emphasizes the support of metacognitive processes—planning, monitoring, and evaluating own thinking—which research in mathematics education identifies as key to effective learning and independent problem-solving (Hidayat et al., 2025; Sercenia & Prudente, 2023). Furthermore, research shows that metacognitive approaches involving reflective challenges, self-monitoring, and the regulation of cognitive processes significantly improve students' mathematical performance, problem-solving ability, and metacognitive skills (Young & Worrell, 2018). The intervention described later in this article lies at the intersection of mathematical and pedagogical interventions; therefore, in the following text, we will use the uniform term *intervention*, which refers to a pedagogical intervention in mathematics.

Strategies for Solving Mathematical Problems

Since our study also deals with strategies for solving mathematical problems, we present a theoretical definition of this issue. According to Budínová (2018), several strategies for solving mathematical problems, which we list below, can be distinguished based on their sophistication:

(1) *unguided experiment (trial and error method)*–the student experimentally tests the given relationships, gradually approaching the result, but does not look for connections between the individual solutions that would allow them to speed up the solution;

(2) *guided experiment*–the student experimentally tests the given relationships, after a few steps notices a certain pattern and uses it to speed up the solution;

(3) *arithmetic solution with graphical representation*–the student uses graphical representation to find mutual relationships and solves the problem numerically;

(4) *arithmetic solution without graphical representation*–the student finds relationships between variables without a picture, i.e., their arithmetic concepts are at a higher level of development; and

(5) *algebraic solution*–the student correctly identifies the unknowns, constructs an equation or system of equations, and solves it correctly.

These strategies are part of a broader framework of heuristic approaches, which Budínová (2018) describes as key to the development of mathematical thinking and problem-solving skills.

Stages of Solving Word Problems in Mathematics

In the Czech educational context, the process of solving word problems in mathematics is typically divided into five stages. These can be adapted depending on the nature, structure, and difficulty of the task (Pólya, 1945):

1. *Task analysis*–consists of clarifying and finding relationships between the given data and the subject of the question. An effective analysis leads the solver to the appropriate choice of arithmetic operations, which lead to the mathematization of the real situation and finding the result of the word problem.
2. *Mathematization of the real-life situation* is the logical outcome of the analysis, which consists of writing down the corresponding mathematical expression (arithmetic example, equation, or system of equations).
3. *Solving a mathematically formulated problem* consists of solving the arithmetic example, equation, or system of equations.
4. *The answer* to the problem leads the student to compare the result with the conditions of the problem.
5. *Verification of correctness* confirms that the result found is the correct solution to the problem and corresponds to its specified conditions.

Case Study of a Mathematically Gifted Student

The present case study describes an intervention with a student formally identified as mathematically gifted. It focuses on the manifestations of his mathematical giftedness and provides a detailed account of the course and outcomes of a year-long intervention designed to expand his problem-solving strategies and develop his metacognition. The study also draws on a semi-structured interview in which the student expressed his need for individual development in mathematics–a need that had not been met in his school environment. His inclusion in the year-long intervention was based precisely on this need.

METHODOLOGY

Given the nature of the research problem and its aims, a qualitative research strategy was adopted. The design selected was a case study, which, as Baxter and Jack (2008) note, is appropriate when the researcher seeks to understand a specific case in depth and within its full context, while also reflecting the participant's experience from their own perspective.

Research Participants

The participant was selected using a combination of volunteer sampling, based on his willingness to take part in the research (Murairwa, 2015), and convenience sampling, based on accessibility (Campbell et al., 2020). This approach is typical in qualitative research, which prioritizes depth of understanding and contextual insight over representativeness. At the time of the study, the participant was fourteen, and later fifteen years old–a student in the eighth and subsequently ninth year of a small rural lower secondary school. At the age of thirteen, he had been formally diagnosed as mathematically gifted by an educational Psychological Counselling Center (PCC). Another participant in the research was the student's mother (a special education teacher), who had long observed various manifestations of her son's giftedness for mathematics. It was the student's mother who initiated her son's assessment at the PCC, because his mathematical abilities had not been recognized in the school environment, let alone systematically developed.

Ethical Considerations

The study was conducted in accordance with ethical principles for research involving human participants. The participants were informed about the aims and procedures of the study prior to participation, and participation was voluntary. In the case of a minor participant, written informed consent was obtained from their legal guardian before participation in the study. All collected data were anonymized to ensure the protection of personal data in accordance with ethical principles of educational research and applicable legislation.

Data Collection Methods

At the beginning of the research (January 2024), a semi-structured interview was conducted with the participants. The interview questions were developed by the authors and formulated with regard to the objectives of the research survey. The semi-structured interview with the student lasted around 30-40 minutes and was initially focused on his perception of mathematics lessons at school. He was asked to describe what a typical mathematics lesson in his class looked like and to outline the kinds of tasks usually given. He was then asked about his personal perspective on his mathematical development. He was asked questions such as:

1. What does a typical mathematics lesson look like in your class?
2. How do you feel about mathematics lessons?
3. What do you do during a mathematics lesson once you've finished your work?
4. Do you discuss the solutions to the problems with your teacher? If so, describe how.
5. What do you enjoy about mathematics?
6. What are your future plans for personal development in mathematics?

The semi-structured interview with the mother lasted 20-30 minutes. The mother was asked about the observed manifestations of her son's mathematical giftedness in preschool and later school age and about the possible existence of phenomena limiting the development of her son's giftedness. The mother was asked questions such as:

1. Did your son show an interest in mathematical concepts during his preschool years? If so, please describe which ones.
2. How did your son's mathematical abilities and skills develop during his preschool years?
3. How did your son's mathematical abilities and skills develop during his early years of schooling?

4. Are there any factors influencing the development of your son's mathematical gift in the school environment? If so, please describe them.

Both interviews were recorded with the participant's consent, transcribed verbatim, and subsequently analyzed.

From January to December 2024, the student was also observed directly during a series of intervention sessions. This intervention consisted of two phases (initial and follow-up). There were 15 intervention sessions with the student (totaling 30 hours), held at two-week intervals. The primary observation tool during the intervention was field notes (Yin, 2018). The notes included reflective records by the researchers. The researchers' initial intention was to record the entire course of the intervention, but the research participants did not agree to this, so no videos were taken during the observation. Interviews with the research participants and observations of the student were conducted in his home environment. Each session in both phases of the intervention lasted approximately 2 hours (1 hour of task completion and 1 hour of discussion).

The initial phase of the intervention took place during the first two sessions and monitored the student's manifestations of mathematical giftedness in accordance with Renzulli's (1978) *three-ring model*, modified for mathematical giftedness (Schindler & Rott, 2017). In this context, it focused on the student's preferred strategies for solving assigned mathematical problems. The tool used to observe signs of giftedness was worksheets containing non-standard mathematical problems, which the student solved independently.

The follow-up phase of the intervention took place over the course of 13 additional sessions (totaling 26 hours) and again consisted of assigning worksheets containing non-standard mathematical problems. Once again, the student first solved the problems independently and then reflected on his solutions with the researchers. During this process, the researchers observed phenomena that played a significant role in the development of individual components of metacognition (Flavell, 1979; Lai et al., 2015; Straka, 2021; Veenman et al., 2006).

Additional data included his written work (completed worksheets), which were analyzed, along with the reports and recommendations from the PCC.

Data Analysis

All the student's personal data has been pseudonymized, making it impossible to identify him. For the purposes of this case study, we will refer to the student by the pseudonym Thomas. The analysis of interview data was conducted using an inductive coding approach without the use of software. Data from observations and the content analysis of the student's worksheets were based on a deductive coding approach, which utilized codes corresponding to manifestations of giftedness described in the professional literature according to Renzulli's (1978) *three-ring model*, modified for mathematical giftedness (Schindler & Rott, 2017).

Interview transcripts were first analyzed using open coding, which was conducted independently by two researchers. Subsequently, the codes created and their meanings were compared and discussed to achieve consensus in the interpretation of the data. This procedure served as a check for agreement between the coders and contributed to increasing the reliability of the analysis. Data from recommendations and reports from the PCC and students' written work with solutions to mathematical problems were subjected to content analysis of the text. To increase the credibility of the results, a member checking procedure was also used, in which participants were familiarized with the interpretation of selected parts of the data. The overall research design was designed as a case study, which allows for a deeper understanding of the phenomenon under investigation in its natural context.

Through analysis of the collected data, the researchers identified key manifestations of mathematical giftedness during problem-solving and phenomena related to metacognitive development throughout the intervention.

FINDINGS

Findings From the Interview with the Mother

The mother of a student who worked as a special education teacher at a primary school described her son's long-term interests in a semi-structured interview with researchers. As a child, his interests centered on

Lego, board games, dinosaurs, and toy trains. He also showed an early fascination with numbers and enjoyed counting, though his strongest and most sustained interest was in nature, particularly animals. When he started school, his enthusiasm for numbers and numerical operations persisted. He became increasingly engaged with mathematical problems, especially logic puzzles, riddles, and ciphers. In the primary years, his curiosity about nature deepened, and he developed a new passion for geography. During lower secondary school, these interests continued, and his engagement in activities related to numbers and numerical and logical tasks deepened. Based on Thomas' interests, his mother also noticed manifestations of his mathematical giftedness in comparison with his peers. During an interview with researchers, she mentioned that his giftedness had not been identified by the school environment, let alone systematically developed. These facts led her (not the school) arranged for psychological assessment at the PCC.

Findings From the Psychological Counselling Center

Following an assessment at the PCC when Thomas was 13 years old, it was determined that his current level of overall intellectual performance was very advanced ('very advanced' indicates performance well above age-level expectations, reflecting highly developed skills compared to his peers)–corresponding to the range of exceptional giftedness[1]. For Thomas's further mathematical education in a school environment, the PCC proposed teaching methods in which he would be given creative tasks with ambiguous solutions, differentiated tasks with the option of choosing more demanding and advanced tasks of various types that would deepen his creativity. The PCC also recommended assigning puzzles, brain teasers, and games to develop new strategies. Some recommendations provided by the PCC for the further development of Thomas's mathematical education were systematically used by researchers in creating tasks for both phases of individual intervention.

Findings From the Interview With the Student

In the interview, Thomas described a typical mathematics lesson in his class. He pointed out the teacher's rather unfriendly approach toward the students and the frequent conflicts that occurred during lessons. At the same time, however, he appreciated her clear way of explaining the material, which suited him personally. He describes the atmosphere in class as follows:

> "The teacher comes in, works through the most difficult problems, argues with the students; the kids don't want to write in their exercise books, so she gets angry. She scolds them for just copying from the board and not thinking while they write. It's a confrontational environment; the teacher argues with the students. Everyone is working on the same problems. I understand the teacher's explanations, but the other students don't."

Overall, Thomas has a rather contradictory view of the lessons. He particularly appreciates the fact that the teacher assigns tasks of varying difficulty and introduces students to different problem-solving methods:

> "I like that the teacher gives us different types of tasks, prepares us for all possible scenarios, that she doesn't just give us the easiest ones, but also the more complex ones sometimes, and that she tends to teach the more complex methods rather than the easier ones."

He therefore views it positively that the instruction is not focused only on the simplest tasks but also includes more challenging problems and methods that can foster a deeper understanding of mathematics.

In situations where Thomas has finished the assigned work, he continues independently in class with additional problems from the textbook of his own choosing. However, these problems are no longer systematically checked by the teacher. Thomas comments on this:

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[1] Within the framework of this study, giftedness is understood in accordance with the diagnostic practice of educational and PCCs in the Czech Republic, which define it as a significantly above-average level of cognitive abilities identified using standardized psycho-diagnostic tests, supplemented by other professional methods (IQ tests, cognitive potential tests, specific ability tests, observation, medical history, interviews, and information from the school). Based on the results of the diagnosis, the PCC provides a summary report with a qualitative description of the student's mathematical thinking and recommendations for further work with the student. A student who was diagnosed as gifted on the basis of this standardized diagnosis at the PCC was included in this research study.

> "... if I've finished the problems the teacher assigned to the others, I'm supposed to continue from the textbook and work on whatever I choose myself. I work on my own; the teacher doesn't check my problems. I check them myself based on the results or with my classmates to see how we did and if we got the same answer."

His statement suggests that individual feedback from the teacher is limited, which is why Thomas uses cooperation with his classmates to check his results.

He is particularly critical of the frequent repetition of the lesson material. He feels that too much time is devoted to it, even though, in his view, the students who would need the repetition the most do not actively participate in it:

> "... but I don't like that we spend a lot of time repeating things. We wouldn't need so much repetition ... those who should be reviewing aren't doing it anyway. It bothers everyone that we're reviewing."

He therefore perceives the repetition as tedious and feels that some of his classmates share this opinion.

Other criticisms concern the overall classroom atmosphere. Thomas feels the teacher lacks friendliness and willingness to help, and the lessons seem rather boring to him:

> "... she's a boring teacher; lessons with her are uninteresting. As for extra math problems, I'll do whatever I'm given, but I don't get any sense of satisfaction from it. It would be nice if she gave me more challenging problems. There aren't enough interesting problems."

Although he sometimes receives extra problems, solving them does not provide him with significant motivation or a sense of satisfaction. He would prefer more difficult or interesting problems that would present a greater intellectual challenge for him.

Thomas would also welcome more group work and a more open approach to discussing solutions:

> "... and above all, I personally mind the lack of group work and the teacher's resistance to discussing the results of assignments."

He therefore fulfils his need for sharing and discussion primarily with his classmates. At the same time, he mentions the limited space for his own problem-solving strategies:

> "I don't discuss solutions with the teacher. I discuss them with my classmates. In mathematics, I've given up on a lot of things; I just have to get through it without causing a disturbance. I accept what the teacher tells me, but reluctantly. I solve problems the way the teacher wants, not the way I see fit."

In his opinion, it is precisely collaborative work and sharing methods that could liven up the lessons and contribute to a deeper understanding of the material being taught. From Thomas perspective, maths lessons can be characterized as follows: he appreciates clear explanations and the occasional assignment of more challenging types of problems. At the same time, he mentions a conflicting classroom environment, frequent repetition of material, a lack of more interesting and challenging problems, and limited opportunities for discussion and collaboration among students.

In view of these facts, he was asked the question, "What opportunities do you see for your own development in mathematics?" In response, he expressed his interest and desire for more in-depth individual education in this area. From the school's point of view, this possibility of an individual approach to Thomas's education was not offered, as the school did not take the reports and recommendations from the PCC into account, so he and his mother turned to researchers themselves with a request for pedagogical intervention in mathematics. This intervention was to focus on solving non-standard, advanced, and developmental mathematical tasks that were in line with the PCC's recommendations.

Individual Pedagogical Intervention with the Mathematically Gifted Student

Given the lack of enriching, non-standard mathematical tasks in the school curriculum, and Thomas's personal interest in deeper individualized mathematical education, he was included in a year-long pedagogical intervention in mathematics. The researchers began the intervention with Thomas in January 2024; it consisted of two phases (initial and follow-up) and included a total of 15 consecutive sessions; each held every two weeks. At the time of the intervention, Thomas was an eighth-grade student in lower secondary school.

Initial Phase of the Intervention

The aim of the initial phase of the intervention was to identify patterns related to how a gifted student approaches non-standard mathematical problems and based on these, to observe manifestations of his mathematical giftedness. This phase served as a diagnostic function.

The initial phase of the intervention took place during two meetings between Thomas and the researchers, each lasting 2 hours. During these sessions, Thomas was given non-standard mathematical problems that went beyond the scope of the current grade's mathematics curriculum. Thomas first solved the problems on his own (1 hour) and then, in a discussion with the researchers (1 hour), described his solution process. To identify manifestations of Thomas's mathematical giftedness, the researchers, as part of their observations during the initial phase of the intervention, relied on predefined indicators of giftedness described in the professional literature according to Renzulli's (1978) *three-ring model* modified for mathematical giftedness (Schindler & Rott, 2017). An analysis of his procedures and results in solving the problems assigned during the initial phase of the intervention, as well as discussions with researchers and observations of Thomas while he was solving the problems, revealed phenomena that are categorized below into three areas based on various dimensions of the gifted student's cognitive and metacognitive abilities:

1. *Problem-solving strategies and metacognition*
    - Dominant use of controlled or uncontrolled experimental strategies
    - Frequent reporting of results only, without explanation of the reasoning process, and many problems were solved mentally
    - Difficulties articulating the reasoning process verbally
    - Inattentive reading of task instructions
    - Often no written answer to the question posed
    - No verification of the result
    - Tendency to arrive quickly and intuitively at correct answers, but without reflecting on the process
    - No written construction process for geometry-based tasks
2. *Cognitive abilities and mathematical skills*
    - Logical and analytical thinking
    - Exceptional numerical memory
    - Good computational skills
    - Ability to solve complex problems without prior instruction
    - Occasional minor errors in mental arithmetic
3. *Motivation and perseverance*
    - Ability to maintain focus over the long term
    - Perseverance in solving difficult problems
    - Engagement with mathematical problems
    - Intrinsic motivation and enjoyment of working with mathematical content
    - Solving problems outside of lessons

The phenomena observed during the initial phase of the intervention, some of which are related to specific manifestations of Thomas's mathematical giftedness, are documented below in selected examples of problem solutions.

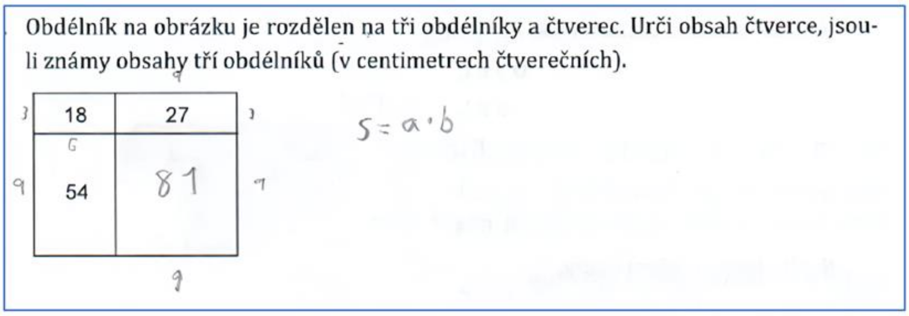


Figure 2. Example of a problem solved mentally using an experimental approach (Source: Anonymised student's work; informed consent obtained)

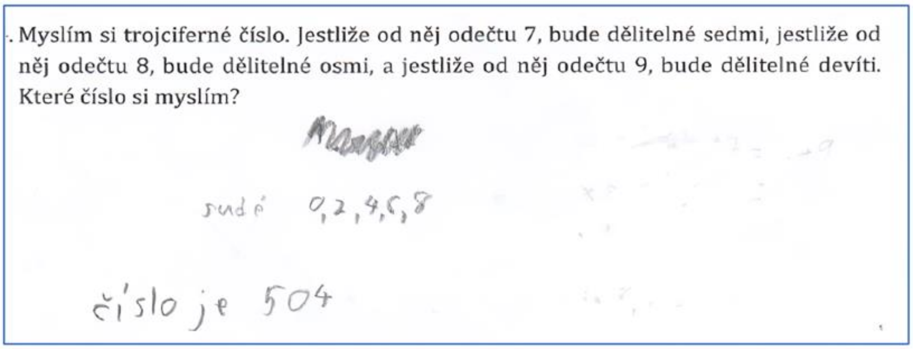


Figure 3. Problem solved mentally (Source: Anonymised student's work; informed consent obtained)

Many of Thomas's early solutions looked like those shown in **Figure 2** and **Figure 3**, where only the final answer was written without any indication of the reasoning process. **Figure 2**, for example, represents the solution of a geometry problem "The rectangle in the diagram is divided into three rectangles and a square. Determine the area of the square if the areas of the three rectangles (in cm²) are known." In this solution, only the result is recorded using the formula for the area of a rectangle. Without further explanation, it is impossible to determine the specific strategy used. Only through subsequent discussion did the researchers discover that Thomas had employed a controlled experimental strategy, demonstrating his exceptional numerical memory and logical-analytical reasoning. He solved the problem correctly, rapidly, and intuitively, but without reflecting on the process.

Figure 3 shows another mentally solved problem "I am thinking of a three-digit number. If I subtract 7 from it, the result is divisible by 7. If I subtract 8, it is divisible by 8, and if I subtract 9, it is divisible by 9. What number am I thinking of?" Thomas noted only his initial idea–identifying an even number as the result–but did not record any further reasoning. The correctness of the answer "The number is 504." confirmed later in discussion, again demonstrated his exceptional numerical memory and logical and analytical thinking, although he did not check his result.

The example solution to the task "The older brother is one quarter older than the younger. In 24 years, the younger brother will be twice as old as the older brother is now. How old are the two brothers?" in **Figure 4** illustrates Thomas's use of an uncontrolled experimental strategy. He systematically wrote down possible ages of two brothers that satisfied the first condition of the task, yet he did not find the correct solution.

It can be stated that many findings obtained on the basis of the analysis of procedures and results of tasks assigned in the initial phase of the intervention, discussions with researchers, and observations of Thomas while solving tasks correspond to the phenomena recorded in Renzulli's (1978) *three-ring model*, which has been modified for mathematical giftedness (Schindler & Rott, 2017). According to this model, Thomas's (1) *above-average mathematical abilities* were confirmed, as evidenced by: the ability to solve challenging problems without prior instructions, an excellent numerical memory, and logical-analytical thinking; (3) *task engagement in applying mathematical thinking*, which is demonstrated by: perseverance in solving difficult problems, enthusiasm for mathematical tasks, internal motivation, and enjoyment of working with mathematical content, as well as solving problems outside of lessons.

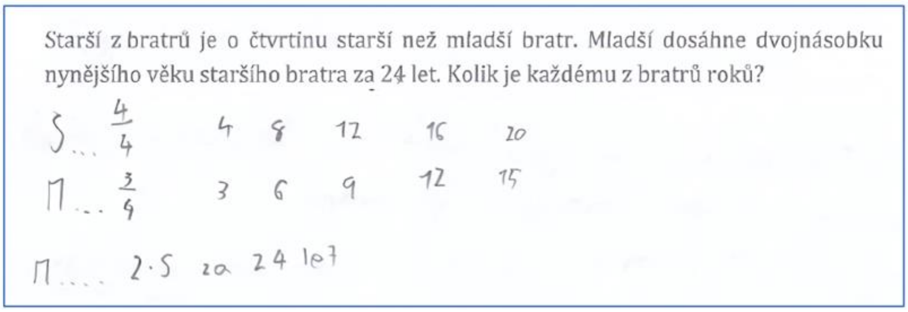


Figure 4. Problem solved using experimental trial and error (Source: Anonymised student's work; informed consent obtained)

Follow-Up Phase of the Intervention

The aim of the follow-up phase of the intervention was to develop metacognition in a mathematically gifted student through non-standard problems that were assigned to him and through discussions with researchers about possible strategies for solving them.

Findings from observations during the initial phase of the intervention, based on an analysis of Thomas's problem-solving procedures and discussions with researchers, led to the design of the follow-up phase of the intervention (13 sessions/26 hours). Given its goal of developing Thomas's metacognition according to the proposed three-component model (Flavell, 1979; Lai et al., 2015; Straka, 2021; Veenman et al., 2006), the follow-up phase of the intervention monitored the following specific phenomena during problem-solving:

- Clarification of the recording of individual steps in solving the task
- Use of appropriate graphic representation of the situation
- Ability to verbally describe the solution to a task
- Verification of the correctness of the task result
- Recording of the verbal answer to the assignment question
- Working with errors
- Expanding the range of solution strategies beyond the experiment
- Ability to generalize knowledge and create new hypotheses or strategies

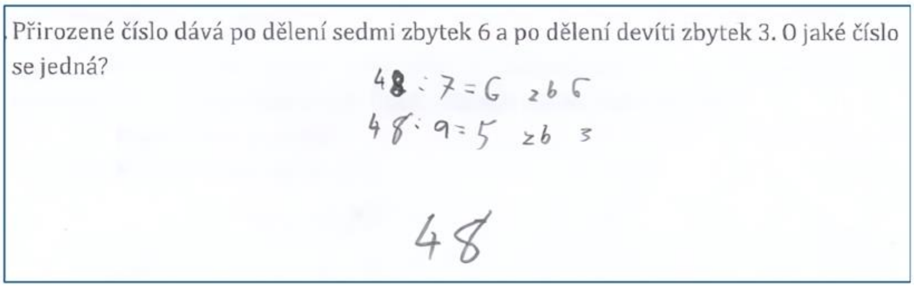
The tool used to achieve the aims of the follow-up phase of the intervention, in accordance with the phenomena under study, was a set of worksheets containing non-standard mathematical tasks. For each worksheet, problems were selected from various areas of school mathematics (fractions, percentages, combinatorics, arithmetic, geometric geometry, divisibility of natural numbers) beyond the scope of the current grade's curriculum. The difficulty of the problems within a given topic gradually increased across the worksheets. The problems were assigned to Thomas based on careful selection to ensure that the learning pathways aligned with the observed metacognitive elements. In selecting them, the researchers drew inspiration from various collections of problems for gifted students, maths competitions such as Matematický klokan (Maths Kangaroo[2]) or Matematická olympiáda (Maths Olympics[3]), and problems from Cermat[4] tests, which cognitively meet the needs of gifted students.

It should be noted that before the intervention period, Thomas was not familiar with algebraic methods for solving problems using equations or systems of equations. The assigned tasks were designed to foster Thomas's creativity in solving them, encouraging him to build on his own experiences and actively construct

[2] Details about the Czech version of the competition, current dates, and the history of previous years in the Czech Republic can be found on the website https://www.matematickyklokan.net.

3 Current information about the Czech version of the competition and the history of previous years can be found on the website https://www.matematickaolympiada.cz/.

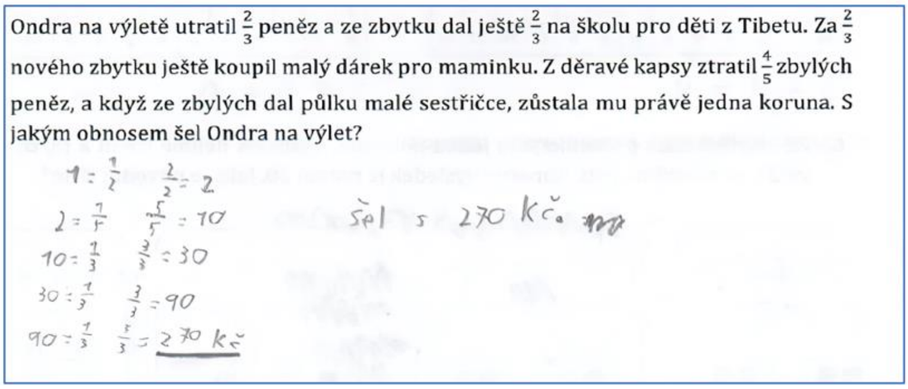
[4] Information about CERMAT, a state-funded organization that provides and develops systems for assessing educational outcomes, is available on the website https://cermat.gov.cz/.


Přirozené číslo dává po dělení sedmi zbytek 6 a po dělení devíti zbytek 3. O jaké číslo se jedná?

48 : 7 = 6 zb 6
48 : 9 = 5 zb 3

48

Figure 5. Task solved by memory experiment with verification of the correctness of the result (Source: Anonymised student's work; informed consent obtained)


Ondra na výletě utratil $\frac{2}{3}$ peněz a ze zbytku dal ještě $\frac{2}{3}$ na školu pro děti z Tibetu. Za $\frac{2}{3}$ nového zbytku ještě koupil malý dárek pro maminku. Z děravé kapsy ztratil $\frac{4}{5}$ zbylých peněz, a když ze zbylých dal půlku malé sestřičce, zůstala mu právě jedna koruna. S jakým obnosem šel Ondra na výlet?

1 = $\frac{1}{2}$ $\frac{2}{2}$ = 2
2 = $\frac{1}{5}$ $\frac{5}{5}$ = 10
10 = $\frac{1}{3}$ $\frac{3}{3}$ = 30
30 = $\frac{1}{3}$ $\frac{3}{3}$ = 90
90 = $\frac{1}{3}$ $\frac{3}{3}$ = 270 kč

šel s 270 kč.

Figure 6. Solution to the problem with a description of each step of the procedure (Source: Anonymised student's work; informed consent obtained)

his own knowledge, and required the selection of solution strategies. Thomas first solved the assigned problems independently and then reflected on the steps of his procedures during a discussion with the researchers. During the discussion, the researchers supported Thomas in his verbal explanations of the procedures used and guided him toward mastering the skill of adequately recording the individual steps of problem-solving. They emphasized the creation of his own insights, while during reflection they prematurely introduced more advanced material focused primarily on algebraic solution strategies and monitored whether Thomas used them in subsequent tasks. The researchers encouraged Thomas to use appropriate graphical representations of the given real-life situations. Error handling also played a key role in the discussion. The researchers were interested in observing whether previous reflections had led Thomas to acquire new solution methods, primarily algebraic ones, to the ability to record individual solution steps meaningfully and correctly, to eliminate errors he had made in previous tasks, and to other phenomena identified above that are signs of metacognitive development (NRC, 2000; Straka, 2021). Therefore, to help Thomas become aware of metacognitive processes, the researchers asked him questions during the discussion such as: "What approach did you take when solving the problem?" or "In what other way could the problem be solved differently?" In cases where Thomas made a mistake while solving the problem, the researchers used the question "Where might you have gone wrong?" to guide him toward recognizing the causes of his failure and, in the context of metacognition, toward understanding his own thought process.

The commented examples of solutions to assigned tasks in the follow-up phase of the intervention provide a record of the observed phenomena with regard to the development of metacognition in a mathematically gifted student.

Figure 5 shows an example of a task "A natural number gives a remainder of 6 when divided by seven and a remainder of 3 when divided by nine. What number is it?" solved by a memory-controlled experiment (when reflecting on the task, Thomas verbally described that he first considered the individual multiples of seven, to which he added the number six, and then divided this sum by nine and observed the remainder after this division). Thomas's reasoning led him to correct answer, but he then verified its correctness with a calculation.

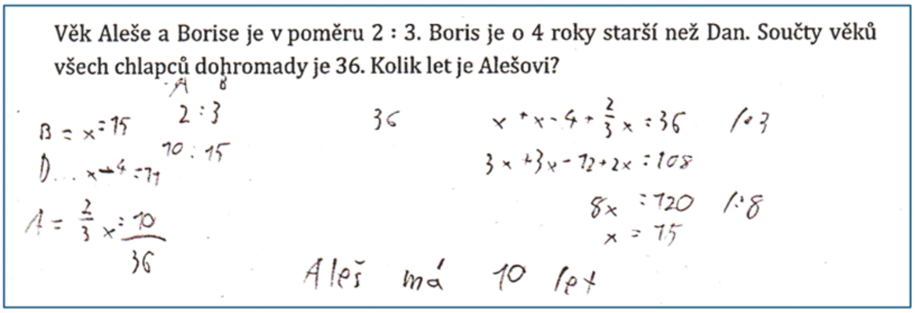


Figure 7. Task solved algebraically (Source: Anonymised student's work; informed consent obtained)

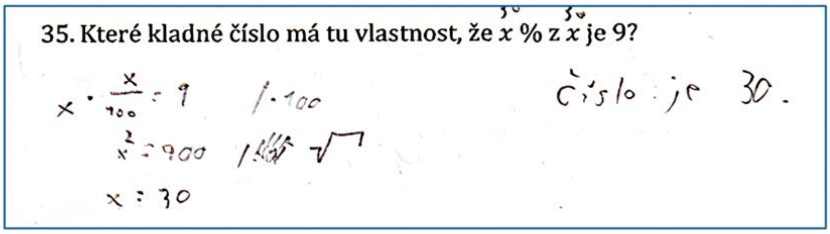


Figure 8. Problem solved algebraically (Source: Anonymised student's work; informed consent obtained)

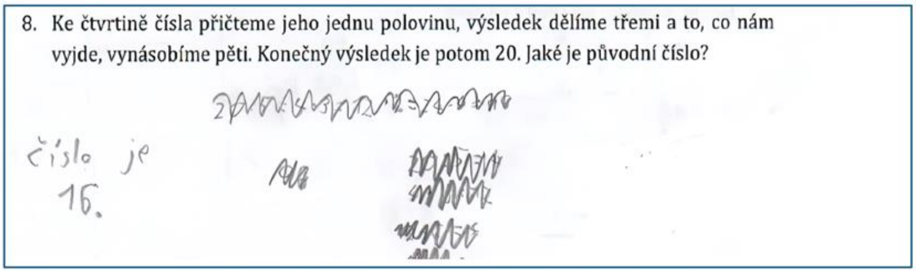


Figure 9. Solution to task no. 8 in the initial phase of the intervention (Source: Anonymised student's work; informed consent obtained)

Figure 6 shows an example of a student's solution to the task "Ondra spent $\frac{2}{3}$of his money on a trip and gave another $\frac{2}{3}$of the rest to a school for children in Tibet. He used $\frac{2}{3}$of the new remainder to buy a small gift for his mother. He lost $\frac{4}{5}$ of the remaining money from his pocket, which had a hole in it, and when he gave half of it to his sister, he was left with one crown. How much money did Ondra take on the trip?" Thomas's solution records the correct result, which now includes a record of the individual steps, including the answer "He took 270 CZK." Note that this record is not mathematically correct.

Figure 7 shows the algebraic method used by the student to solve the problem "The ages of Aleš and Boris are in a ratio of 2:3. Boris is 4 years older than Dan. The sum of the ages of all the boys is 36. How old is Aleš?" with the correct result and answer "Aleš is 10 years old." Thomas chose Boris's age as the variable x and, according to the conditions of the problem, correctly wrote down the linear equation, which he then solved using equivalent transformations.

The example in **Figure 8** shows the algebraic solution to the problem "Which positive number has the property that *x*% of *x* is 9?" This approach led Thomas to a quadratic equation, which he solved. He wrote the correct answer "The number is 30." The methods used to solve the problems in **Figure 7** and **Figure 8** demonstrate Thomas's ability to quickly learn new mathematical concepts and expand his range of solution strategies.

Researchers observed a visible shift in Thomas during the follow-up phase of the intervention, when they gave him the same or similar types of tasks as in the initial phase. This shift is illustrated, for example, by task number 8 "Add half of a number to a quarter of that number, divide the result by three, and multiply the result by five. The result is 20. What is the original number?" In the initial phase of the intervention, Thomas

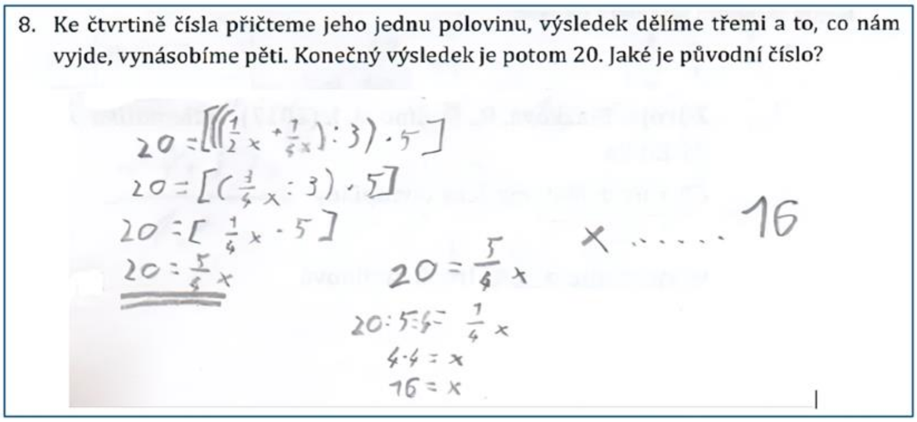


Figure 10. Task number 8 in the follow-up phase of the intervention solved algebraically (Source: Anonymised student's work; informed consent obtained)

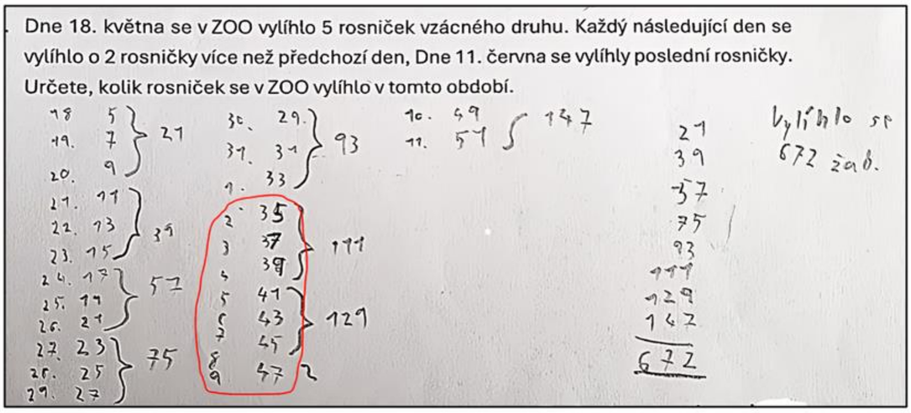


Figure 11. Solution to the arithmetic sequence problem (Source: Anonymised student's work; informed consent obtained)

solved the task experimentally, as indicated by the crossed-out calculation records, and then wrote down only the correct result "The number is 16" (**Figure 9**). Two months later, Thomas solved the same task again, this time choosing an algebraic method using a linear equation (**Figure 10**). This approach demonstrates Thomas's ability to learn new mathematical concepts and use new strategies.

Another interesting phenomenon in the development of metacognition occurred in two tasks in **Figure 11** and **Figure 12**. These are arithmetic sequence tasks. At the beginning of the follow-up phase of the intervention, Thomas solved the given task "On May 18, five rare tree frogs hatched at the zoo. Each subsequent day, 2 more tree frogs hatched than the previous day. On June 11, the last tree frogs hatched. Determine how many tree frogs hatched during this period" (**Figure 11**) by writing down the numbers of hatched tree frogs for each day in the given period. He then added up all the numbers in groups of three and added these partial sums using a written algorithm. He supplemented his solution with an answer "672 frogs hatched." This method of solving the problem is not very effective, and he also made a mistake during the calculation, highlighted in red in **Figure 11**, where he only wrote down the numbers of tree frogs seven times for the days from June 2 to June 9 (a total of eight days). Discussing this solution with researchers led Thomas to find the error and its cause, and he was offered a more efficient method of solving the problem instead of using partial sums.

Thomas solved a problem like the one in **Figure 11** two months later. It was a task "On March 1, two snowdrops bloomed in the garden. Each subsequent day, 5 more snowdrops bloomed in the garden than the previous day. On March 15, the last snowdrops bloomed. Determine how many snowdrops bloomed in the garden during this period" (**Figure 12**). While solving it, he realized that he had solved a similar problem before (**Figure 11**). He wrote down the number of blooming snowdrops for individual days. This time, however, he did not calculate partial sums of all snowdrops, as in the task in **Figure 11**, but noticed the relationship of

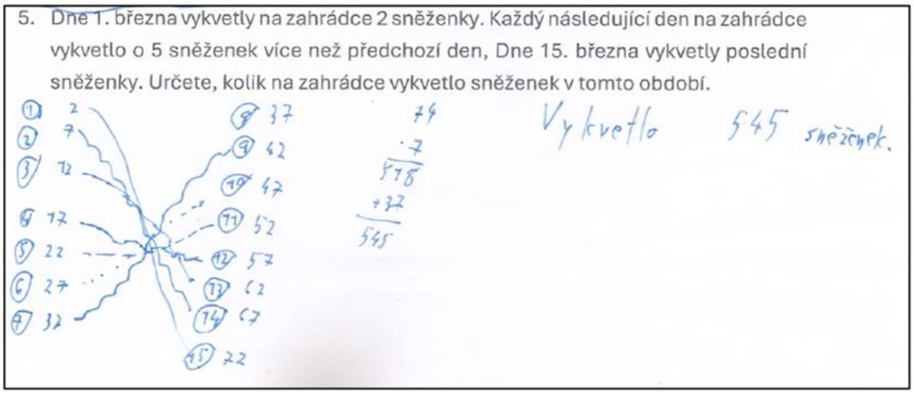


Figure 12. Solving an arithmetic sequence problem (Source: Anonymised student's work; informed consent obtained)

equality between the sums of snowdrops on the first and last days, the second and penultimate days, etc. This relationship is shown in **Figure 12** by lines connecting two addends with a sum of 74 (the number of snowdrops blooming on the first and last days together, etc.). To determine the number of all snowdrops blooming between March 1 and March 15, Thomas used multiplication. This method represents an original approach to solving the problem, the ability to generalize acquired knowledge, and create new strategies. Note that in the final calculation, Thomas made a numerical error when adding in the written algorithm (he wrote 518 + 37 = 545), so the result of the problem written in the answer "545 snowdrops bloomed." is not correct.

During the follow-up phase of the intervention, which monitored the phenomena described above based on methods and procedures for solving non-standard mathematical problems and subsequent verbal reflection, Thomas demonstrated significant development in all components of metacognition. The interconnection between the observed phenomena and metacognition, in accordance with its three-component model (Flavell, 1979; Lai et al., 2015; Straka, 2021; Veenman et al., 2006), will be described in detail in the discussion section of this article.

DISCUSSION

The findings of the present study demonstrate that, at the conclusion of a one-year intervention with Thomas, a mathematically gifted student, significant changes and development occurred in all components of his metacognition. In accordance with the three-component model of metacognition (Flavell, 1979; Lai et al., 2015; Straka, 2021; Veenman et al., 2006), Thomas developed his metacognitive knowledge through the effective storage of new knowledge and its connection to existing knowledge. By applying the acquired knowledge in new situations (**Figure 7**, **Figure 8**, **Figure 10**, and **Figure 12**) and by using new (primarily algebraic) strategies (**Figure 7**, **Figure 8**, and **Figure 10**), he developed his metacognitive regulation. Verifying the correctness of the result and using appropriate graphical representations for planning the solution process and managing his thinking are further regulatory steps newly emerging in Thomas in terms of metacognition. Metacognitive insight occurred during the process of solving the task in **Figure 12**, when he realized that he had already solved a task of a similar type. He actively responded to this insight by selecting a more effective strategy than in the previous similar task, which is an important moment for the development of metacognitive regulation (Noor, 2022; Veenman et al., 2006; Zepeda & Nokes-Malach, 2023). Verbal descriptions of problem-solving procedures after task completion, which were based on precise mathematical expressions, sound argumentation, and justification, demonstrate changes in the area of metacognitive knowledge. Thomas began to symbolically record his problem-solving procedures so that he could better track his thinking and check his calculation steps, which, according to Veenman et al. (2006), relates to metacognitive regulation.

Searching for and finding errors when a problem yielded an incorrect result (**Figure 11** and **Figure 12**) prompted him to recognize the causes of his failure and to understand his own thought process, which represents the two metacognitive components of regulation and monitoring (Flavell, 1979; Veenman et al., 2006). The examples shown in **Figure 3** and **Figure 9** capture the moment when cognitive conflicts occurred during the solution process, as Thomas, while checking his work, encountered an intermediate result that did not match his intuitive expectations. He realized his mistake, crossed out the original calculation, reorganized his thinking, and chose a different strategy. These facts were documented in the subsequent discussion with the researchers. The described handling of the error describes the activation of metacognitive monitoring through the activation of cognitive conflict, a phenomenon described in the professional literature by Flavell (1979). During the observation, while Thomas was solving the assigned tasks, the researchers observed his frequent verification of his understanding of the task instructions, ongoing verification of the effectiveness of the chosen strategies, and checking of intermediate results, which are examples of developing metacognitive monitoring (Efklides, 2014; Flavell, 1979; Veenman et al., 2006). An interesting phenomenon in metacognitive regulation was the recording of the answer to a question from the assigned task (**Figure 6**, **Figure 7**, **Figure 8**, **Figure 9**, **Figure 11**, and **Figure 12**), during which, in a discussion with the researchers, Thomas stated that it helped him clarify the meaning of the result in relation to the context of the task.

In an interview after a year of intervention, Thomas also mentioned that in maths classes many of his classmates do not understand the teacher's explanations, which relate to new material or problem-solving procedures. A very important point he made in this interview is that Thomas often began to explain new topics, including how to solve assigned problems, to his classmates using various problem-solving strategies. We consider this moment to be a significant manifestation of the development of his metacognitive awareness.

Based on the findings described above, which were observed during the intervention process, we can confirm the presence of characteristics in Thomas that correspond to aspects of Renzulli's (1978) *three-ring model*, as modified for mathematical giftedness (Schindler & Rott, 2017):

1. *Above-average mathematical abilities*–logical-analytical thinking (**Figure 2**, **Figure 3**, **Figure 9**, **Figure 11**, and **Figure 12**), rapid mastery of mathematical concepts (**Figure 7**, **Figure 8**, **Figure 10**, and **Figure 12**), excellent numerical memory, ability to solve problems without prior instruction.
2. *Creativity and intuition*–he demonstrated original approaches to solving problems (**Figure 9** and **Figure 12**), generalized newly acquired knowledge, and developed new strategies (**Figure 12**).
3. *Engagement in the application of mathematical thinking*–persistence in solving difficult problems, enthusiasm for mathematical tasks, intrinsic motivation, and enjoyment of working with mathematical content, solving problems outside of class.

Research findings confirm that pedagogical intervention can help students develop a deeper awareness of their own thinking, develop their understanding of cognitive processes, and increase the effectiveness of problem-solving strategies among mathematically gifted students, all of which provide opportunities for these individuals to fully realize their potential (Erdoğan, 2025; Pintrich, 2002; Schraw & Graham, 1997; Young & Worrell, 2018).

The findings of the one-year intervention with Thomas also correspond with the results of research described in the professional literature (NRC, 2000; Straka, 2021), namely that discussing possible strategies for solving a task with a student and providing pedagogical intervention (Knox, 2017; Portešová & Veenman, 2021) can strengthen their metacognitive knowledge.

Limitations of the Study

The research results are limited due to the uniqueness of the observed case, as they were obtained from only one case study of a mathematically gifted student who showed significant development of metacognition through individual intervention. Nevertheless, we consider the findings of this research to be an important message for parents of mathematically gifted students and for educators who have the opportunity to work with them.

Although the study describes the course and impact of a one-year intervention, it should be borne in mind that the development of the student's metacognition may also have been influenced by other factors, in particular the school environment and other educational experiences of the student.

CONCLUSION

The study presented addresses a topic that deserves attention in school practice. It focuses on a mathematically gifted student in lower secondary school who has been diagnosed with giftedness in mathematics. Despite a professional examination at the PCC and its recommendation for an individual approach to the student's education in mathematics, his mathematical giftedness was not recognized in the school environment, the PCC's recommendations were not accepted by the teachers, and the student ceased to develop in this area at school. In an interview with researchers, he stated that he was bored in mathematics lessons. He found the tasks he was given in mathematics class uninteresting. Moreover, they were the same tasks that his classmates were given at the same time. In his words, his time was wasted in mathematics classes, although he still enjoyed it. It can be assumed that this persistent situation could lead to stagnation and a loss of motivation and interest in the subject. The student was aware of this fact and took the initiative to develop his maths skills outside of school. The researchers offered him an intervention at home focused on mathematics, in which he solved non-standard tasks beyond the scope of the mathematics curriculum for his current grade. During the intervention, a few important phenomena were observed in terms of the students' metacognitive development in mathematics, which would probably not have occurred under the prevailing conditions at school.

This study draws attention to the fact that there are a number of children whose giftedness has not been identified in the school environment and who, despite confirmation of their giftedness and recommendations from the PCC for further education, have not been treated as gifted. Given these circumstances, these students may experience boredom, frustration, or loss of motivation at school. In this context, we are looking for ways to prevent these factors and offer these students further development within the school environment. The key approach to teaching mathematically gifted students is differentiation and individualization of teaching, which aims to provide each student with adequate space, including gifted students. This teaching may include the creation of enriching activities that support the development of giftedness. We then recommend assigning differentiated maths problems to gifted students, dividing them into three levels of difficulty and allowing them to choose more challenging versions of the problems, offering them additional problems of various types beyond the scope of the standard maths curriculum (logical puzzles, brain teasers, games for developing various strategies). To support the development of creativity, gifted students can be given tasks with ambiguous solutions or multiple solutions, or they can be invited to create their own tasks, or they can be asked to present the results of their activities to the class and guided to create their own portfolio. Another option for developing giftedness is to involve students in various competitions and Olympiads, or collaborate on the creation of teaching materials for classmates, etc. Individual intervention with gifted students, one of which we have described in this case study, can also be considered a tool for developing mathematical giftedness.

While exact numbers of mathematically gifted students in schools are not available, the topic addresses a globally recognized need to support gifted learners. This study, which describes the course of an individualized pedagogical intervention with a mathematically gifted student and documents significant changes in the student's metacognitive development following the intervention, may serve as a useful resource for educators working with mathematically gifted students. Both phases of the intervention described, which significantly benefited the observed student in terms of metacognitive development, can serve as a methodological guide for researchers studying the metacognitive processes of mathematically gifted students.

Author contributions: JP: conceptualization, data curation, formal analysis, methodology, writing–original draft, and writing–review & editing & **JV:** data curation, methodology, writing–original draft, and writing–review & editing. Both authors approved the final version of the article.

Funding: This article was supported by the Faculty of Education, Masaryk University, Czech Republic.

Acknowledgments: The authors would like to thank the Faculty of Education, Masaryk University, Czech Republic, for institutional support.

Ethics declaration: The authors stated that no ethical approval was required for this study.

AI statement: During the preparation of this work, the authors used ChatGPT (OpenAI) in order to improve language and readability. After using this tool, the authors reviewed and edited the content as needed and took full responsibility for the content of the published article.

Declaration of interest: The authors declared no competing interests.

Data availability: Data generated or analyzed during this study are available from the authors on request.

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