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Research Article

Pre-service mathematics teachers' understanding of conditional probability in the context of the COVID-19 pandemic

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ABSTRACT

During the last two years, the COVID-19 pandemic had a secondary effect of increased media content loaded with mathematical, often probabilistic information (and misinformation). Our exploratory study investigates the probabilistic intuitions, misconceptions, biases, and fallacies in conditional probability reasoning of mathematics teacher candidates in the context of the pandemic. The pre-service mathematics teachers who participated in our study were given a questionnaire with five contextual conditional probability problems, all formulated similarly to media statements often encountered when discussing the COVID-19 pandemic. Our findings confirm the previous findings on biases and fallacies related to conditional probability problems with a social context. They were also indicative of several types of errors (both numerical and logical) as more common than expected. Our results also reveal that pre-service mathematics teachers apparently separate the content learned in the classroom from the application of the knowledge in critical examination of the information to which they are daily exposed by the media.

Keywords: conditional probability, context of a probability problem, pre-service mathematics teachers, COVID-19 pandemic, media in teaching of probability and statistics

INTRODUCTION

One of the important purposes of mathematics education is to teach individuals how to make rational, informed choices, and to critically examine information met in daily and professional life (Díaz & Batanero, 2008; Kus & Cakiroglu, 2020; Watson, 1997). The need of overcoming deterministic thinking and accepting the role of chance in various aspects of life in the modern "information society" results in the increased importance of probability and statistics for all citizens. This led to the introduction of probability and statistics in most countries' curricula during the last few decades (Batanero et al., 2016; Estrada et al., 2018; Kus & Cakiroglu, 2020).

One of the four key constructs regarding probabilistic thinking is conditional probability, the other three being sample space, probability of an event, and probability comparisons (Jones et al., 1997). Conditional probability, including Bayesian reasoning, is fundamentally connected to how individuals deal with uncertainty, and how they use additional information to adjust their initial probability estimates. It helps individuals to understand possible risks and make adequate decisions in everyday and professional life (Díaz & Batanero, 2008; Estrada et al., 2018; Gal, 2002). This makes it an essential component of statistical literacy, highly relevant in medicine, law, education, psychology, and other professional fields (Batanero et al., 2016; Díaz & Batanero, 2009; Díaz & Fuente, 2007).

However, even if conditional probability appears in numerous contexts, it is often considered to be an especially hard probabilistic notion. Feller (1968, p. 114) said: "The notion of conditional probability is a basic tool of probability theory, and it is unfortunate that its great simplicity is somewhat obscured by a singularly clumsy terminology". Problems with understanding of conditional probability have been noted and studied, and the lack of intuition about conditional probability is also notable in the historical development of probability (Díaz & Batanero, 2008). Furthermore, as Greer (2001) noted, probabilistic thinking in general appears counter-intuitive in many cases due to the cultural bias towards deterministic thinking. These are some of the reasons why in many, probably most, curricula the teaching of conditional probability is limited to senior secondary level (in Croatia, e.g., it is only introduced in the 12th year of schooling), even if some of its aspects could successfully be introduced somewhat earlier (Watson, 1995).

The role of general mathematical, and particularly statistical, literacy has been further emphasized lately. The onset of the SARS-CoV/COVID-19 pandemic brought additional exposure to information involving probabilistic thinking (Muñiz-Rodríguez et al., 2020), due to the wide coverage of the pandemic in popular media (television, radio, printed media, news portals, and social media). The fact that the pandemic resulted in a surplus of mathematical data and statements in popular media can be viewed both as a challenge and an opportunity for all mathematics educators, especially since using media in teaching and discussing probability is a well-known idea (Jones, 2005; Ozen, 2013; Watson, 1997).

Sound fundamentals in (conditional) probability are necessary to understand probabilistic and statistical content embedded in a wider social context, to correctly interpret conditional statements from media, to make correct inferences from the published data, to detect fallacies in media statements. In many cases intuitive reasoning can be sufficient for critically reading media statements involving probabilistic claims, and besides, in most daily situations precise analysis and calculation are both superfluous and impractical. Developing adequate probabilistic intuitions is one of the basic components of teaching probability, and the development of probabilistic thinking depends on the interaction between "intuition, logical development, and the effects of formal instructions" (Greer, 2001, p. 25). Previous research suggests that systematic probabilistic instruction can have a positive effect on some intuitive biases (Díaz & Batanero, 2008; Fischbein & Gazit, 1984). In contrast, it was also noted that students, even those who reached the highest (numerical) level of probabilistic reasoning according to the Tarr and Jones (1997) framework, tend to engage in subjective reasoning and have various misconceptions, particularly when dealing with probability problems having a social context (Carles & Huerta, 2007; Olgun & Isiksal-Bostan, 2019). This becomes especially important when teachers are concerned, since "If teachers have the same misconceptions as their students, how can they develop appropriate lessons and tasks to facilitate students' understanding of conditional probability?" (Stohl, 2005, p. 352).

Thus, our goal is to explore the level of probabilistic intuitions and the presence of biases and misconceptions amongst pre-service mathematics teachers when dealing with contextual conditional probability (COVID-19 related) information present in popular media. More specifically, we aim to investigate the following question: How do pre-service secondary school mathematics teachers' approach, deconstruct, critically examine, and interpret information from media involving conditional probability? We also hope to contribute to existing research on conditional probability fallacies, and to future development of better ways of teaching the hard, but important topic of conditional probability.

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

A fair amount of research on probability reasoning and probabilistic misconceptions has been conducted lately, but not much has dealt specifically with pre-service mathematics teachers' understanding of conditional probability. Among the existing studies on adults' (including pre-service mathematics teachers') difficulties in understanding conditional probability, we highlight the studies by Díaz and Batanero (2009) and Díaz and Fuente (2007). Díaz and Fuente (2007) discussed the apparent uncorrelation between student's formal knowledge and their psychological biases. Díaz and Batanero (2009) found that even after instruction some biases could not be overcome.

Existing research identified several reasons for learning difficulties in probability: students' negative attitudes, lack of reasoning skills, readiness level, misconceptions, teacher, and age (Bursali & Ozdemir, 2019). Further research on the attitudes towards probability and statistics, and their impact on teaching and learning, suggests the importance of teachers' positive attitudes for successful teaching (not only) of probability and statistics (Estrada & Batanero, 2020; Estrada et al., 2018). Together with emotions and beliefs, attitudes belong to the affective domain in mathematics education (Gómez-Chacón, 2000; Philipp, 2007). They can act as a bridge between the other two components of the domain, and they can include them (Di Martino & Zan, 2015). Attitudes have a stronger cognitive component than emotions and are relatively stable over time (Estrada & Batanero, 2020). Ruz et al. (2021) found a positive association between content knowledge and attitudes of pre-service teachers, while Segarra and Julià (2022) also confirmed that teachers' efficacy beliefs as well as attitudes towards mathematics are key factors for pre-service teacher's mathematics academic achievement.

Attitudes have an impact on the development of intuitions. As Fischbein and Gazit (1984, p. 2) noted, "new intuitive attitudes can be developed only through the personal involvement of the learner in a practical activity. Intuitions (cognitive beliefs) cannot be modified by verbal explanations only." By intuition they mean "basically, a global, synthetic, non-explicitly justified evaluation or prediction. Such a global cognition is felt by the subject as being self-evident, self-consistent, and hardly questionable" (Fischbein & Gazit, 1984, p. 2). Borovcnik and Peard (1996) noted that in contrast to other branches of mathematics, in probability counterintuitive results are found even at very elementary levels, particularly frequently when dealing with conditional probability. This results in several psychological biases and erroneous intuitions related to conditional probability (Batanero et al., 2016; Díaz & Batanero, 2009; Jones, 2005).

Tarr and Jones (1997) described a framework for assessing (middle school students') thinking in conditional probability (and independence). This framework consists of four levels of thinking about conditional probability: At level 1, the subjective level, students ignore the given numerical information in making predictions, and use subjective reasoning when considering the conditional probability of an event. At level 2, the transitional level, students demonstrate incomplete recognition of whether consecutive events are related or not, and their use of numbers to determine conditional probability is inappropriate. At level 3, the informal quantitative level, students can (imprecisely) quantify conditional probabilities and keep track of the composition of the sample space, and they recognize the change of probabilities in "without replacement" situations in contrast to the "with replacement" situations. Finally, at level 4, the numerical level, students assign correct numerical values to conditional probabilities, state the necessary conditions under which two events are related, and use numerical reasoning to compare the probabilities before and after a trial. Díaz and Fuente (2007) developed a general questionnaire to globally assess formal understanding of conditional probability and the psychological biases and difficulties related to this concept. Three of these are relevant for the present study.

The first is the *confusion of the inverse* (CI), also known as the fallacy of the transposed conditional. It refers to confusing $P(A|B)$, the probability of event A given that event B has occurred, with its inverse $P(B|A)$ (Falk, 1986). In a medical context it is often encountered as confusing the sensitivity of a test with its positive predictive value (PPV). The sensitivity is a property of a diagnostic test that if a medical condition which is being tested for is present in the tested individual, then the test will give a positive result: sensitivity = P(positive test|medical condition present). The probability P(test negative|medical condition not present) is called specificity. The PPV is the inverse conditional probability of sensitivity, i.e., the probability that an individual who tested positive really has the condition: PPV=P(medical condition present|positive test). In contrast to sensitivity, PPV depends not only on the test itself and how it is performed (the 'active' side), but also on the prevalence of the condition, i.e., on the ratio of all individuals having the condition in the tested population (the 'passive' side of the test). The CI is also related to a well-known psychological difficulty, the perception of $P(A|B)$ regarding causality. If B is perceived as a possible cause of A, $P(A|B)$ is viewed as a causal relation, while if A is perceived as a possible cause of B, $P(A|B)$ is viewed as a diagnostic relation (Batanero et al., 2016; Díaz & Fuente, 2007; Tversky & Kahnemann, 1982).

The second fallacy we consider in our study is the *base-rate fallacy* (BRF). This refers to ignoring the base-rate (the ratio of the considered feature in the whole population) when calculating or estimating a conditional probability. To use the children-riddle example from Bar-Hillel (1983, p. 39), when answering "Why is more grass consumed by white sheep than by black sheep?", BRF is committed if one concludes that a white sheep eats more grass than a black one. If this can be concluded or not, with a smaller or higher degree of certainty, depends on the base-rate (the ratio of white sheep among all sheep), and possibly other factors. In a medical context, the BRF typically appears as ignoring prevalence when estimating or calculating PPV (Bar-Hillel, 1983).

The third fallacy relevant for our study is the *confusion of joint and conditional probability* (CJC). It refers to using $P(A \& B)$, probability of both A and B occurring, as $P(A|B)$. This type of error was noted in several previous studies, including those with pre-service teachers (Díaz & Fuente, 2007; Estrada & Batanero, 2006). In a medical context we may encounter this, e. g., if, knowing that a disease is common to 2% of males, but 3% of females, in a town with 4500 male and 5500 female inhabitants, one concludes that the probability that a person diagnosed with the disease is male is calculated as 2%·4500/10000 = 0.9 % instead of 2%·4500/(2%·4500+3%·5500) ≈ 35%.

These, as well as other known fallacies, seem to be more prominent when problems have a context related to everyday experiences and information. It was confirmed that even students with high level probabilistic proficiency in "classical" probability problems tend to focus on their beliefs and everyday experiences when confronted with problems with a social or health related context (Greer, 2001; Olgun & Isiksal-Bostan, 2019).

Contextual conditional probability problems often tend to be presented as applications, after the topic has been taught formally. In contrast, Freudenthal (1983) suggested that teaching of conditional probability might begin with problems which let students make explorations of phenomena involved in the topic; then the formal teaching of conditional probability would become a means of organizing these phenomena. This means departing from concrete, realistic situations to be modelled, and using problem solving in context (Carles & Huerta, 2007; Lonjedo Vicent et al., 2012; Nilsson et al., 2014). By context we mean a "particular situation in which problems are put forward. In a context, a particular concept such as conditional probability has a specific meaning or is used with a specific sense" (Carles & Huerta, 2007, p. 703). Problems having the same context can be further classified by phenomena (Carles & Huerta, 2007). A particularly well-known context is the "diagnostic test context", exploring the conditional probability of (not) being infected (or pregnant, or having consumed a drug) if a medical test had a positive, or negative, result. We concentrate on several medical contexts (the diagnostic test context, the context of vaccination, and the context of medical status of hospital patients), all referring to phenomena related to the current COVID-19 pandemic.

The target group of our study are pre-service mathematics teachers because they will soon teach future generations about conditional probability. Also, they are expected to have a higher level of understanding of these topics and should more easily recognize the possible pitfalls than the average citizen. We explore their conditional probability intuitions, misconceptions and biases, and usage of their formal conditional probability knowledge in the context of the pandemic as presented in popular media. Thus, our study contributes to studies on misconceptions, fallacies and biases present in conditional probability reasoning, and the influence of context (Carles & Huerta, 2007; Olgun & Isiksal-Bostan, 2019), as well as to studies on intuitions (Fischbein & Schnarch, 1997). Furthermore, it also relates to existing studies on pre-service mathematics teachers' understanding of probability (Batanero et al., 2015; Bursali & Ozdemir, 2019; Estrada & Batanero, 2006; Goméz-Torres et al., 2016; Hokor, 2020; Ozen, 2013) and to studies on using media in teaching probability and statistics (Kus & Cakiroglu, 2020; Ozen, 2013; Watson, 1997). We also aim to obtain some new ideas for development of the future teaching of conditional probability, since "If one investigates the student's difficulties and misconceptions, one does not identify only logical deficiencies. One identifies, very often, intuitive tendencies, intuitive interpretations, and models–tacit or conscious–that contradict the formal knowledge with which school tries to endow the student" (Fischbein, 1999, p. 49).

METHOD

Preparation of the Study

Our study was designed as a pre-announced, anonymous 15-minute questionnaire with contextual conditional probability problems. Due to the short time planned for the execution of the study, we initially limited the total number of problems to at most five, with at most one open-ended problem. Our initial formulation of 17 problems was based on media statements in Croatian popular media regarding the pandemic. These were discussed among professionals from Croatian scientific institutions (two pure mathematicians, two mathematics educators, and two chemists to ensure the real-life relevance of the problems). Then six problems were discarded (one for being trivial, four because they involved equivalent solution techniques to other four problems in the list, and one because the correct formulation would require stating too much additional information).

To reduce the number of problems to five, we compared them, regarding solution techniques, tested probability content and relations to biases, to the questionnaire developed by Díaz and Fuente (2007). Several of their problems were found not to be related to our problems. In particular, we did not compare our problems with those of their items having a diachronic setting (involving series of experiments), theoretical questions on conditional probability and classical "with (or without) replacement" problems. We made the final selection and formulation of our problems as follows: Our problem 1 is fully equivalent to Díaz and Fuente's (2007) item 1 (computing joint and conditional probabilities from a two-way table in a synchronic setting). Our problem 2 is formulated to resemble (regarding mathematical content and detectable biases) to their item 2 (in both cases the correct solution involves using the Bayes' formula, and two distractors are mathematically equivalent in both problems); our problem 2 also has some similarities with their item 5 (medical context, calculation of conditional probability, one distractor equivalent regarding detectable bias). The final formulation of our problem 3 was modelled after their item 7 (to assess the presence of CI and distinguishing of causal and diagnostic situations), but with additional numerical information (to make it clear that the probability of one event is smaller than the probability of the other). Our only open-ended problem 4 was formulated to have the same solution procedure (based on Bayes' and total probability calculations) as their item 18. Finally, our problem 5 has no related item in the Díaz and Fuente's (2007) questionnaire. It is a problem with minimal changes of the vague original text from an online magazine. The reason for including this problem was to test the intuitive skills of interpreting probabilistic information from media, since "once students with some rudimentary statistical concepts in hand are exposed to the media, a second need–to read and interpret written reports, rather than just perform computations–becomes important." (Watson, 1997).

Before the main study, a pilot questionnaire was administered to 26 volunteers (1st year undergraduate students of chemistry). They were chosen for three reasons. First, they (like all 1st year undergraduate students in Croatia) had recently, in their last year of secondary schooling, been introduced to the topic. Secondly, they also understand real-life applications, so they were expected to detect possible content-related errors. Finally, at the time of the pilot the COVID-19 restrictions reduced the numbers of attendees at classes in our faculty, but more chemistry than mathematics students were attending lectures, thus providing a larger pool for the pilot. After the pilot, two problems needed to be slightly rephrased to become clearer; no other changes were made.

The reasons for the short time given to solve the problems are as follows. One was to ensure as many participants as possible, since previous experience showed that the students of our department are more likely to agree to participate in short research. Also, as our goal was to assess the level of intuition and critical thinking skills when confronted with media texts involving conditional probability, we wanted to detect if they can quickly recognize and estimate correct conclusions. Furthermore, when reading a media text, it is not likely that anyone would (regularly) perform detailed calculations to check the claims, but it is important to understand and critically examine the given (and missing) information, and to notice fallacies. It was also expected that, since statements like the ones in our problems are widely present in Croatian popular media for almost two years, the participants had on some occasions given thought to such statements. Finally, all 26 participants of the pilot survey handed their solutions in before the 15 minutes deadline, even if none left blank answers in more than one problem.

Participants

The participants of our study were students of three graduate educational mathematical study programs (4th and 5th year of university education) at the largest department of mathematics in Croatia. These are mathematically intense programs, intended for future mathematics teachers in secondary school. The participants were all of the students who were present in two lectures of obligatory courses (due to the COVID-19 epidemic this was less than all students enrolled: 51/168, i.e., 30% of all students of our educational

Table 1. Problem 1 in the questionnaire

Age	COVID-19	Only other illnesses	Total
Under 60	2	7	9
60+	18	42	60
Total	20	49	69

graduate studies participated in the questionnaire, the others were not attending live lectures at the time for various reasons). All of them had been introduced to conditional probability in the last year of pre-university education. They further studied it in their last semester of undergraduate studies (i.e., either half a year, or one and a half years before their participation in our study) in the obligatory course "Probability and statistics" (its syllabus includes conditional probability and Bayesian reasoning, and in scope of the course they have met problems equivalent to our problems 1 to 4 with various contexts).

Content of the Questionnaire

All our five problems had a synchronic (static) setting, and a COVID-19-related context. Two asked for precise numerical information (no. 1 and 2), while the other three could also be solved by intuition or estimation (no. 3, 4, and 5). Problems no. 2, 3, and 4 were what Carles and Huerta (2007) as well as Lonjedo Vicent et al. (2012) name ternary problems in conditional probability[1]. As said previously, our first four problems have related items (regarding the method of solution and biases they assess) in Díaz and Fuente's (2007) questionnaire.

Our *problem 1* requires calculating three probabilities from a two-way table with absolute frequencies. *In a hospital the current state is as given in* **Table 1**.

If a patient from this hospital is randomly selected, answer the following questions:

a. *What is the probability that the selected patient has COVID*-19?

b. *If the selected patient has* 60 *years or more, what is the probability of this person to be infected with COVID*-19?

c. *If the selected patient is known to have COVID*-19, *what is the probability of this person being* 60 *years or older*?

The correct answers to problem 1 can be read from **Table 1**: The probability in 1a is 20/69≈29% (this was the only non-conditional probability problem in the questionnaire). The probability in 1b is 18/60=30%, and in 1c the correct solution is 18/20=90%. As possible errors and fallacies to be detected by this problem we expected that in some cases participants would give the same solution for 1b and 1c, which would be indicative of CI, or write 18/69 in either 1b or 1c (CJC).

Our *problem 2* is a contextual multiple-choice Bayesian problem. It was not specified that the hospitalization or vaccination was for COVID-19, as the same reasoning applies to any illness, even to vaccinations against one and hospitalization because of a different reason.

The ratio of vaccinated among the hospitalized persons is 30%. *The ratio of vaccinated persons in the total population is* 50%. *Of the total population,* 0.1% *is or was hospitalized. The probability of a vaccinated person to be hospitalized is, as follows:*

a. 30/50=0.6=60%,

b. 30%,

c. 30/50×0.1%=0.06%,

d. 0.1%×30%=0.001×0.3=0.0003=0.03%, and

e. *None of the offered solutions is correct*.

[1] Ternary problems in conditional probability satisfy three conditions: "(1) One conditional probability is involved, either with known or unknown data or both; (2) Three probabilities are known; (3) All probabilities, both known and unknown, are connected by ternary relationships." (Lonjedo Vicent et al., 2012, p. 323). Ternary relationships are algebraic identities connecting three probabilities, for example $P(A)+P(\text{not } A)=1$.

Table 2. A tabular approach to problem 4 in the questionnaire

	Tests positive	Tests negative	Total
Infected	Sensitivity times prevalence=1.6%	2%-1.6%=0.4%	Prevalence: 2%
Not infected	98%-97.02%=0.98%	Specificity times 98%=97.02%	100%-2%=98%
Total	1.6%+0.98%=2.58%	0.4%+97.02%=97.42%	100%

The correct solution is (c), obtained from the Bayes' formula $P(H|V)=P(V|H)\cdot P(H)/P(V)$ with H denoting "being hospitalized", and V denoting "being vaccinated". The distractors were added to detect (a) BRF, (b) CI, and (d) CJC (in this option $P(H|V)$ is calculated as $P(V|H)\times P(H)=P(V \& H)$).

Our *problem 3* is modelled to assert the presence of the CI fallacy in a COVID-19-vaccinations context:

In the land Probabilistica, only 25% *of the population is vaccinated against "corona". By now,* 1.5% *of its population died from "corona". If we compare the probability p that a person vaccinated against "corona" will die from it, and the probability q that a person who dies from "corona" was vaccinated against it, then for Probabilistica the following is true:*

(a) $p<q$, (b) $p=q$, or (c) $p>q$.

Mathematically, this problem is a version of problem 2, with some data symbolic instead of numerical. The correct solution is (a), corresponding to using the Bayes' formula $p=P(D|V)=P(V|D)\times P(D)/P(V)$="$q$ times a quotient obviously smaller than 1" $<q$ (by D we denote the event of "dying from 'corona'"). This problem was aimed at detecting the prevalence of CI among the participants: If a participant would confuse the inverse conditional probabilities, (s)he would have chosen option (b). The main difference to problem 2 is that here the correct option (a) could be chosen on ground of intuition, or even based on misconceptions and biases. If a participant would use the CJC of BRF fallacy, depending on if (s)he tried to think of p in terms of q or vice versa, (s)he would have chosen (a) or (c). Choosing option (c) might also be a sign of wrong intuition, possibly based on personal beliefs about vaccination, or confusion between causal and diagnostic interpretation of conditional probability.

Our open-ended problem 4 is a Bayes' problem with a diagnostic test context:

The sensitivity of a test (the ratio of truly infected ones among the positive ones) is 80%. *The specificity of a test (the ratio of truly non-infected ones among the negative ones) is* 99%. *The prevalence, i.e., the ratio of infected individuals in the population, is* 2%. *We randomly select a person, test him or her, and the test was positive. What is the probability that this person really is infected? If you don't know how to calculate it precisely, please give an estimate.*

There are two strategies for a correct solution, i.e., calculating PPV=P(infected|tests positive). One is using a two-way table (or a tree-diagram) to represent the data; the other is using Bayes' formula. The tabular solution is as follows: From the data given in the problem, we calculate **Table 2**. From it we read that 1.6% of the population is both infected and tests positive, and in total 2.58% of the population tests positive. By definition of conditional probability, it follows that P(infected|tests positive)=1.6/2.58≈62%.

The solution using Bayes' formula is, as follows:

P(infected|tests positive)=P(tests positive|infected)×P(infected)/P(tests positive)=sensitivity×prevalence/P(+).

The denominator can be calculated using the total probability formula:

P(+)=sensitivity×prevalence+(1-specificity)×(1-prevalence).

Problem 4 is obviously harder and more time-consuming than the previous three, but it explicitly allowed an estimate if exact procedure is not known. Furthermore, it was expected that the previous three problems would give the participants some hints on how to solve problem 4. Regarding fallacies, we expected to detect CI here (if a participant gives 80% as answer), or CJC (in case the product of sensitivity and prevalence, i.e., 1.6%, is given as answer).

Finally, our *problem 5* was added as a problem with a vague formulation, with partially given information, as typically found in popular media:

*The more people are tested for SARS-CoV-*2*/COVID-*19 *(all with the same test, so equal reliability of the test for all tested persons), the more we will be sure how many people in the population are really infected with it, because it will become surer that a positive person is infected. True or not?*

Table 3. Percentages of correct and "blank" solutions

Problem number	1a	1b	1c	2	3	4	5
Correct solutions	49 (96%)	45 (88%)	35 (69%)	17 (33%)	17 (33%)	0 (0%)	0 (0%)
No attempt of solution	0 (0%)	0 (0%)	2 (4%)	2 (4%)	5 (10%)	16 (31%)	6 (12%)

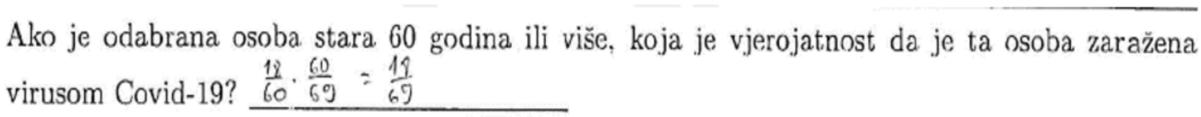
Ako je odabrana osoba stara 60 godina ili više, koja je vjerojatnost da je ta osoba zaražena virusom Covid-19? 12/60 · 60/69 = 12/69

Figure 1. Confusion of joint and conditional probability in problem 1b

a. *Yes, it is true.*

b. *No, it is not true.*

c. *It depends on* ______________________.

As we see, mathematics, in particular conditional probability, is not explicitly present in the formulation. From a mathematical (and contextual) point of view, problem 5 is analogous to problem 4, but it was meant to detect if participants can recognize the relevant unstated information. In this problem it is the role of the base rate (the prevalence of the infection in the tested population, which might also have been hinted to attentive participants by problems 2 and 4).

The real-life significance of problem 5 is that it relates to a common error in thinking that if more people are tested, we always detect more true infections. Obviously, the reliability of a medical test (its sensitivity and specificity, and the expertise and adherence to clinical rules in performing it[2]) affect the absolute and relative frequency of truly infected persons detected by the test. If these were the only factors affecting these frequencies, then by testing more people we would always detect more true cases (in absolute numbers). But, even if the test itself is the same, the ratio of true and false positives depends on the prevalence of the disease in the tested population. This is most easily seen if we calculate the analogue of **Table 2** in the case of prevalence, say, 6%. Then the relative frequency of false positives among all positively tested persons is 0.94/5.74≈16%, while with prevalence 2% the relative frequency of false positives among all positively tested persons is 0.98/2.58≈38%. Generally, for a fixed test the relative frequency of false positives is a decreasing function of prevalence[3]. This is one of the reasons why most countries chose not to test large random samples of their population on COVID-19, but rather tested samples which only include symptomatic persons and contacts of positive ones, thus ensuring a higher prevalence in the tested population. Consequently, in problem 5 both answers (a) and (b) are wrong (and indicative of BRF), while option (c) ("it depends") is only fully correct if prevalence is mentioned as a reason.

RESULTS

As expected, 1a was the problem with most correct solutions (96%, see **Table 3**), followed by 1b (88%) and 1c (69%). Only 32/51 participants (63%) solved both 1b and 1c correctly. Problems 1a and 1b were the only problems which every participant attempted to solve[4]. Two participants showed indications of CI, writing in 1c the same answer as in 1b. Also, three participants exhibited CJC, two in 1b and one in 1c. Of those, one (see **Figure 1**) calculated P(COVID-19|60+) as P(COVID-19|60+)×P(60+)=P(COVID-19 & 60+).

The most frequent error (8/17 incorrect solutions) in 1c were attempts to use various formulas instead of reading the probability directly from the table (**Figure 2**), which resulted in overly complicated, unsuccessful solution attempts. Also, in 4/17 incorrect solutions of 1c, the solution was given as 20/60, i.e., instead of P(60+|COVID-19), they calculated P(COVID-19)/P(60+).

[2] The clinical test results are also affected by biological and pre-analytic factors (Yohe, 2020).

[3] This relative frequency is equal to $x/(x+y)$, where x=(1-specificity)×(1-prevalence) and y=sensitivity×prevalence.

[4] We note that 5/51 (10%) participants equated the exact value 20/69 with the rounded value 0.29 in 1a, not using the approximate equality sign, which indicates their recklessness of the difference between these two numbers.

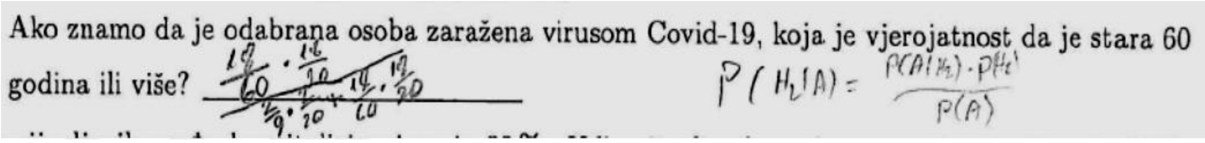
Ako znamo da je odabrana osoba zaražena virusom Covid-19, koja je vjerojatnost da je stara 60 godina ili više?

Figure 2. Confusion regarding conditional and total probability in problem 1c

Table 4. Types of solution attempts in problem 4

Type	O	CI	B	T	CJC	X	Y	I	Other
Frequency	16 (31%)	11 (22%)	10 (20%)	3 (6%)	1 (2%)	3 (6%)	3 (6%)	2 (4%)	2 (4%)

One might consider if the results in problem 1 indicate that it is easier for students to determine a conditional probability from a row (1b) than from a column (1c) in a two-way table, which, if true, could only have psychological reasons. However, problems equivalent to our problem1.b) (conditional probability from a row in a two-way table) and 1c (conditional probability from a column in a two-way table) were also posed by Díaz and Fuentes (2007) and by Estrada and Batanero (2006). Their problems with reading conditional probability from a row were correctly solved by 56% and 52% participants, respectively; and for their problems of reading conditional probabilities from a column they obtained 59% and 56% correct answers, respectively[5]. This suggests that our observed difference in the successes of solving problems 1b and 1c is likely to be a result of chance.

The fact that problem 1 was by far the best solved of the problems also confirms findings from previous research, which suggest that Bayesian computations are simpler when information is given in absolute instead of relative frequencies. For example, Martignon and Wassner (2002) demonstrated the high success rate of applying Bayesian reasoning if it has been taught with the help of absolute frequencies and tree diagrams. Still, Batanero et al. (2015) as well as Estrada and Batanero (2006), found several semiotic conflicts existing among pre-service teachers with understanding frequencies and probabilities even in simple 2×2 two-way tables.

As seen from **Table 3**, in problem 2 only 17/51 (33%) of the participants chose the correct option. Almost a quarter, 12/51 (24%) seem to have had no definite idea how to solve this problem, as they chose option (e) or left the solution area blank. Only 5/51 (10%) chose option (a), so apparently BRF was not a big problem here. No participant chose option (b), corresponding to CI. The CJC-option (d) was chosen by 17/51 (33%) participants, i.e., equally many as have chosen the correct option. Here one must also consider the possibility of personal beliefs (in this case a possible opinion against vaccination) as a reason for choosing (d), where the ratio of all vaccinated persons is ignored (as mentioned before, it is an already by Greer (2011), and Olgun and Isiksal-Bostan (2019) observed phenomenon that students sometimes focus on their beliefs and everyday experiences rather than analytic approaches when confronted with problems with a social context).

Problem 3 was correctly solved by 17/51 (33%) of participants as well. While in problem 2 the CI-option was not chosen by anyone, in problem 3 the CI-option b) was chosen by 25% (13/51 participants). In this problem the confusion about the correct solution was particularly easy to notice, since of the remaining participants 16/51 (31%) chose option (c) and 5/51 (10%) left the solution area blank.

We also note that many showed indications of inconsistent thinking processes. As mentioned in the "Method" section, the correct approach from problem 2 as well as some incorrect ones would result in the correct solution of problem 3. Thus, we expected more correct solutions in problem 3 than in problem 2, however, the fraction was the same, and in fact only 6/51 (12%) of the participants solved both problems 2 and 3 correctly. We conjecture that this was because the data in problem 3 were partly numerical, partly symbolic, and intend to investigate this in a follow-up interview-based investigation.

Since in the pilot-survey only 1/26 participants left the solution area for problem 4 blank, it was a bit surprising to see a much larger frequency, 31%, of blank solutions from students of mathematics. A possible reason, which also needs to be confirmed by additional investigations, is that our participants are by their 4th and 5th year of study more reluctant to risk a wrong attempt, being long trained in precision. Due to the time-

[5] In our pilot study, 65% participants solved 1b correctly, and 69% solved 1c correctly.

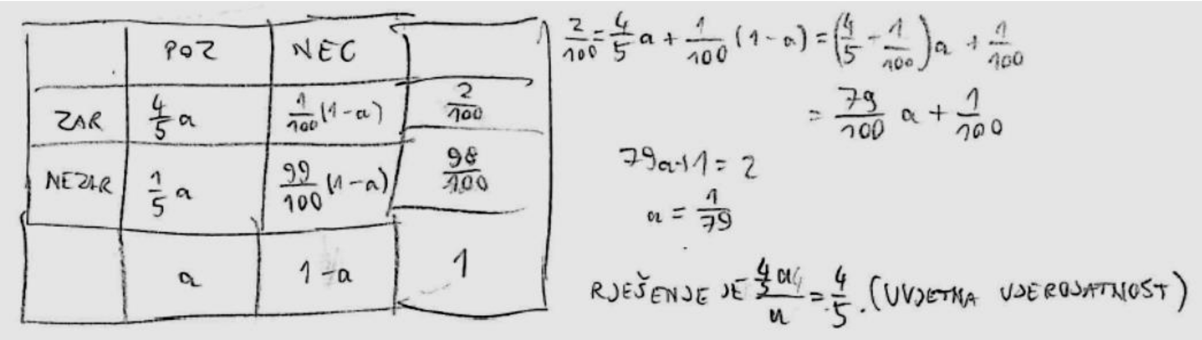

Figure 3. A tabular partially correct solution to problem 4 [ZAR: INF(ected); NEZAR: NOT INF(ected); POZ: POS(itive);NEG: NEG(ative); Rješenje je: The solution is; & Uvjetna vjerojatnost: Conditional probability]

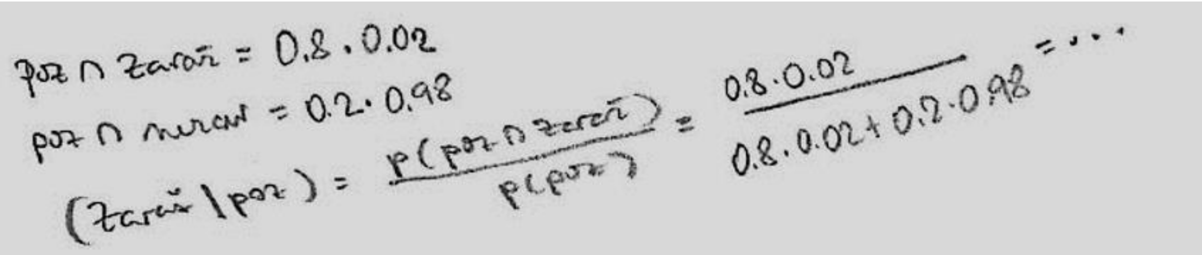

Figure 4. An attempt of using the Bayes' formula in problem 4, but with incorrect calculation of total probability [poz: pos(itive); zaraž: infect(ed); nezar: not inf(ected)]

limit, the fact that no participant fully solved problem 4 was less surprising. Since this was the only open-ended problem, the solution approaches were both plainly readable and more varied, so it is worthwhile to fully classify the solution attempts for this problem (**Table 4**): 0–blank solution; CI–used sensitivity as PPV; B–using Bayes' formula; CJC–multiplied prevalence with sensitivity to obtain 1.6%; T–attempts of tabular or tree-diagram solution; X–result 80.8% or 81%; Y–estimates without explanation, but far from the correct percentage; I–estimates near to the correct value. Class X is particularly interesting, as the solution 80.8% was totally unexpected. One of those three participants who gave this result attempted an explanation of multiplying sensitivity (80%=0.8) by 1.01 to obtain 80.8%, writing "ratio of positive amongst the negative=1.01."

Of the three participants who attempted to solve the problem using a two-way table or three-diagram, two just gave a beginning, not using it further, while one (see **Figure 3**) made a full table and calculation, albeit with an error (prevalence as percentage of infected was correctly used, but then (s)he calculated the number of positive infected ones as sensitivity times total number of positive, instead of sensitivity times prevalence, and made an equivalent error regarding the not infected ones). If we attribute this error to lack of time, we can consider this participant as essentially knowing how to correctly solve the problem.

There were 10 attempts to use the Bayes' formula. One participant miscalculated the total probability in the denominator (**Figure 4**), using 1-P(+|infected)=P(-|infected) for P(+|not infected). Another participant made a somewhat similar error, calculating P(+|not infected)×P(not infected) as 99.98·0.01 (most likely 99.98 is a typo). One participant used the Bayes' formula correctly (albeit unnecessarily) in problem 1c, wrote it correctly next to problem 2, but did not notice the correct conditionality in problem 4. This participant also used sensitivity as P(infected|+), i.e., committed CI here. Another participant wrote the correct Bayes' formula for $P(A|B)$, took A="person infected" and correctly $P(A)$=0.02, but took B="test is negative" with $P(B)$=0.99 concluding that P(not B)=P(test is positive)=0.01. We see that this participant ignored the conditionality in the definition of specificity of the test. Another wrote P(infected|+)=80%, i.e., committed CI, but did not attempt to give that as the final result; instead, (s)he attempted to calculate the probability of being positive using Bayes' formula, setting P(+|infected)=99%, showing even more confusion about conditionality than most. Three participants have set 0.5 for P(+), exhibiting the known equiprobability bias of assuming that if there are two outcomes in the sample space, then these outcomes are equally likely (Jones, 2005). This is also a typical indication of the Tarr and Jones (1997) transitional level and was noted recently in the study of Olgun and Isiksal-Bostan (2019). One participant wrote the correct Bayes' and total probability formulas, but stated

Table 5. Presence on fallacies

Fallacy	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Total (once or twice)
CI	2 (4%)	0 (0%)	13 (25%)	13 (25%)	n/a	21 (41%)
CJC	2 (4%)	17 (33%)	n/a	1 (2%)	n/a	18 (35%)
BRF	n/a	5	n/a	0	29 (57%)	34 (67%)

Table 6. Data on multiple usage of various fallacies in solution attempts

Number of solution attempts with fallacy	0	1	2	3	4	5
Number of participants	7 (14%)	20 (39%)	13 (25%)	9 (18%)	1 (2%)	1 (2%)

there was not enough time, so this one counts as the second participant essentially knowing how to solve the problem. Also, two participants showed a good intuition, writing 60% as an estimate of the probability, so, in total 4/51 (8%) can be considered as proficient enough to correctly evaluate similar statements in media (two based on intuition, one based on table analysis, and one using Bayes' and total probability formulas). We note here that Díaz and Batanero (2008) confirmed that particularly for such open-ended conditional probability problems instruction can significantly improve the students' performance.

17/51 (33%) of all participants chose the correct option (c) in problem 5, but two of them also checked (a) or (b), so their answers are to be considered as irregular. Of the 15/51 (31%) selecting only (c), not one mentioned prevalence or an equivalent term, so we can say that here essentially all participants fell for BRF, even if many understood that the answer is not trivially yes or no. Most of those choosing 5c (13 participants) said "it depends on the reliability" of the test, ignoring the condition stated in the formulation of the problem.

No participant has fully solved problems 4 and 5, i.e., no participant solved more than three problems correctly: 4/51 (8%) participants solved three problems correctly, 15/51 (29%) participants solved two problems correctly, and 15/51 (29%) solved just one problem correctly. The remaining 17/51 (33%) participants partially solved problem 1. Of the four participants who solved the first three problems successfully, one also had correct intuition in problem 4, and one chose option (c) in problem 5 with reason "reliability of the test", so we can consider these two participants as the most successful, and sufficiently competent in rationally interpreting probabilistic media statements.

Finally, we analyze the presence of CI, CJC, and BRF in the solution attempts. If in problem 5 we take only those choosing options (a) or (b) as indicative of BRF (even if, as noted above, essentially all attempts indicate this bias), we obtain the data on the presence of the fallacies as given in **Table 5** and **Table 6**. No participant used the same fallacy in more than two problems. Seven participants committed CI in two out of four possible problems (one in problems 1 and 4, five in problems 3 and 4). Two participants succumbed to CJC in two of three possible problems. No participant showed BRF in more than one problem. On the other hand, most participants (44/51, i.e., 86%) have at least in one problem shown indications at least one of one of the three fallacies CI, CJC and BRF.

DATA AND ITEM ANALYSIS

After we collected the data, we analyzed the response of each participant in each item, considering the completeness of responses in the open-ended problem 4. We graded the results similarly to the grading in Díaz and Fuente (2007). Students were given 1 point per each correct response in problems 1a,1b, 1c, 2, 3, and 5, and 0.5 point in problem 5 if the correct option (c) was chosen, but with a wrong reason. In problem 4 we gave 0 points if the solution area was left blank or the solution attempt was totally wrong, 1 point if the probabilities were correctly identified or some partially correct argumentation was written, 2 points if the inverse probability to be calculated was correctly identified or if the solution was correctly estimated, 3 points if the Bayes' formula or two-way table was correctly written, and 4 points if the solution was fully correct. The differences of our grading with respect to the one in (Díaz & Fuente, 2007) arise due to the incomparability of our problem 5 to their items and due to some unusual, but still partially correct solution attempts for problem 4 submitted by our participants.

Consequently, the maximum possible score was 10 points. The empirical distribution ranged between 1 and 7.5 points with an average value of 3.92, median 4, mode 3, and standard deviation 1.45 (see **Figure 5**).

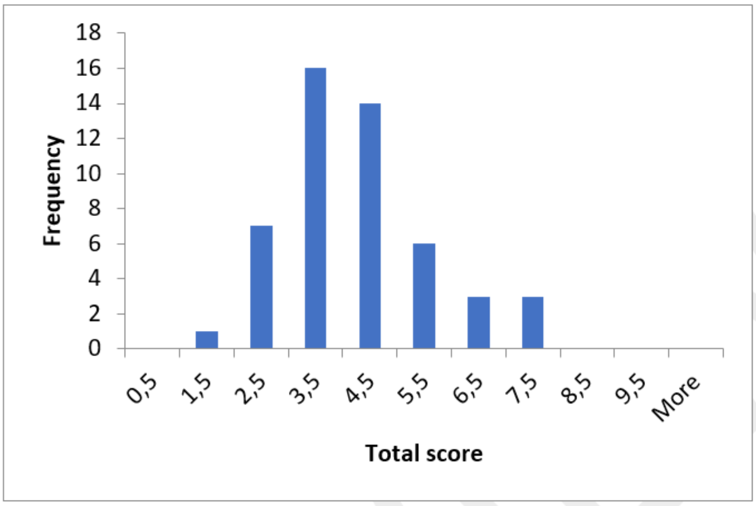

Figure 5. Distribution of the total scores

It is clearly visible that the distribution of scores is skewed to the left, suggesting that the test was relatively hard for the participants.

The data in the result section suggest that problems 1a and 1b were easy for the participants (over 80% correct answers), problem 1c was also relatively easy (69% success), problems 2 and 3 were medium hard, and problems 4 and 5 were hard, without any fully correct solution. Still, in problem 4 12/51 (24%) participants scored 2 or higher, and in problem 5 15/51 (31%) scored 0.5. All our problems had a positive discrimination index (for problems 4 and 5 we took those with half or more points achievable in these problems as successful since there were no fully correct solutions, otherwise their discrimination indices would obviously be 0). We see that the students having better overall scores in general also were more successful in each item.

DISCUSSION AND CONCLUSIONS

While we expected that pre-service teachers of mathematics are fluent enough in conditional probability to notice the common pitfalls often encountered in media statements, our results indicate that it is not necessarily true. A reason of relatively low numbers of correct solutions, particularly in problem 4, is surely the short time given. Furthermore, this was not a formal test meant to be graded, so some students may have been less motivated to exert their knowledge to solve the problems (this is suggested by the relatively high frequencies of blank answers, even in multiple choice questions). However, considering the main purposes of the study, the fact that they met similar problems during their education, and were confronted with the context by media on an almost daily basis for almost two years, the results of our study were not as good as expected. Not a single participant solved all problems correctly, and with the exception of reading conditional probabilities from a two-way table with absolute frequencies (problem 1), the success rate in the problems was 1/3 or less. This indicates an apparent separation of learned content (in this case, conditional probability, and Bayesian calculations) from its usage to check the facts and deconstruct probabilistic misconceptions and fallacies met in daily life. It could be another interesting line of future research to investigate this apparent lack of interest, or possibly mental separation between 'theoretical' learning and 'practical' application thereof, from a psychological as well as a pedagogical standpoint, and to investigate if the participants' attitudes affected these results.

Regarding the three common conditional probability misconceptions, CI, BRF, and CJC, our findings mostly agree with the findings of Díaz and Fuente (2007). They noted that the BRF seems less pervasive than suggested in previous research, which is confirmed by our findings. An exception in our study is problem 5, where all participants ignored the role of base rate, even if they rarely did so in other problems. Due to the problem's formulation, this suggests that when confronted with a real media text, with vague information, even students of mathematics are overstrained and not sufficiently mathematically literate to identify which data is needed to arrive at a correct conclusion. Our findings also confirm Díaz and Fuente's (2007) findings that the different biases in conditional probability reasoning tend to appear unrelated to one another, and sometimes appear opposed or related to some other components of understanding conditional probability problems. Furthermore, our findings are in accordance with Bursali and Ozdemir's (2019) findings. Their research suggested that teachers and pre-service teachers generally successfully identify some types of probabilistic misconceptions, at the same time not noticing other types. All of this relates to a paradox noted by Fischbein (1987), that the intuitions, which are adaptive and useful in organizing and orienting our activities, often give rise to erroneous judgements.

In relation to what Watson (1997) calls a "three-tiered hierarchy" of probabilistic and statistical thinking in social contexts (basic understanding of terminology; understanding of concepts embedded in a wider social context; a questioning attitude applicable to critically examining claims without proper foundation), our results indicate the presence of various problems relating both to the second and third level. To further quote Watson (1997, p. 3):

> "Once students with some rudimentary statistical concepts in hand are exposed to the media, a second need–to read and interpret written reports, rather than just perform computations–becomes important. Some students who have excelled in the traditional symbolic aspects of the mathematics curriculum resist the requirement for reading, interpreting, and writing when mathematics is presented in non-symbolic contexts."

Thus, our study gives further proof of the importance of usage of media texts for teaching and assessing probabilistic and statistical thinking, even without explicit computations, as suggested by Watson (1997).

Besides the trivial confirmation of the necessity of devising better methods of teaching conditional probability in context with the goal of educating citizens mathematically and statistically literate enough to critically evaluate statements met in daily life, several other insights were obtained. One relates to a well-known and often discussed aspect of probability problems, and that is the linguistic ambiguity (and vagueness) of their formulation (Ozen, 2013). As Falk (1989) discussed, in many concrete problems it is often unclear how to extract the relevant information and how to utilize the data needed to compute a conditional probability. Due to the usage of natural language, which is more prominent in contextual problems, students tend to give the same expression a variety of meanings, which further affects their comprehension (Jones, 2005). As our problems were formulated using natural language, our low rates of correct solutions indicate, along with findings of Díaz and Fuente (2007) and Olgun and Isiksal (2019), that linguistic expression is one of the factors influencing understanding and solving of (conditional) probability problems. This impact of vagueness and ambiguity of language on problem-solving is a line of research which surely deserves more future investigations.

Our observations also correspond to other findings of Olgun and Isiksal (2019). They found that a social context puts forward the intuitive component of probabilistic thinking and that many revert to transitional or even subjective reasoning and show context-influenced biases, even if they may have reached the higher informal quantitative or numerical level of the Tarr and Jones (1997) framework. However, our and their findings are obviously dependent on the institutional constraints, since "institution imposes a number of conditions that reshape [a particular knowledge] and "fix" the conditions under which [a certain person] will study [it]" (Chevallard & Sensevy, 2014, p. 39). Even so, these results indicate that more care is needed in teaching (conditional) probability. As in many countries curricular standards suggest that mathematical problem solving should be taught in context (Carles & Huerta, 2007), which is also in accordance with the realistic mathematics education approach, our findings indicate that this should be even more emphasized. To express it concisely, teaching probability "should take into account the characteristics of probability, its multifaceted views (classical, frequentist, subjective or axiomatic probability), common misconceptions, and wrong intuitions" (Estrada et al., 2018, p. 316).

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Ethical declarations: Authors state that all participants of the survey were informed why the research was conducted, their anonymity was assured, and their responses were stored in private files of the researchers. The participation in

the questionnaire was voluntary. For surveys performed anonymously within the Department of Mathematics, University of Zagreb, Croatia, no explicit approval from an ethics committee was required.

Declaration of interest: Authors declare no competing interest.

Data availability: The data upon which this article is founded, and which are summarized in **Table 3**, **Table 4**, **Table 5**, and **Table 6**, were obtained using a questionnaire developed by the authors. **Figure 1**, **Figure 2**, **Figure 3**, and **Figure 4** are anonymous excerpts from participants' answers. The raw data are not publicly available but can be obtained upon reasonable request from the first author.

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